# Log - expo Transformation Flexible Weibull Three Parameter Distribution LET- FW


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# Log - expo Transformation Flexible Weibull Three Parameter Distribution LET- FW

***Dr. Mohamed Mohamed Abdelkader***

**Abstract:**

This paper introduces the LET-FW three parameter distribution. Some mathematical properties of this distribution are studied. Density distribution, Reliability and hazard rate functions are obtained. The ordinary moments, quintile function, mean residual life, Renyi entropy are given. Four methods of estimation of the LET-FW distribution based on complete sampling and the MLE estimates based on censoring type | and || are given. Squared Bias and variances of the estimates via a Mont Carlo simulation study are computed.

**Keyword**: Flexible weibull, LET-FW, Quantile function, Renyi Entropy

## 1- Introduction

The Weibull distribution [19] is often used in the modeling of lifetimes of components of engineering applications, physical systems and many different Felds. In previous years, many authors provided many extensions for the Weibull distribution and their applications. **Mark** (2007) proposed a new two-parameter ageing distribution (flexible Weibull) which is a generalization of the Weibull [12] **M. R. mahmoud** (2011) Based on type II censored sample,the maximum likelihood estimators of the parameters of an extension of the Weibull distribution referred to as the flexible Weibull distribution [11]. **Sinjay** (2013) developed the Bayesian estimation procedure for flexible Weibull distribution under Type-II censoring scheme [17]. **Sinjay** (2015). discussed classical and Bayes estimation procedures for estimating the unknown parameters as well as the reliability and hazard functions of the flexible Weibull distribution [18]. **A.El-Gohary** (2015). introduced a new three parameters model. he called it the exponentiated a flexible Weibull extension (EFW) distribution [1]. **Sangun** (2016). Proposed a general class of fexible Weibull distribution functions which in cludes some well-known modified Weibull distributions [15].

**Abdelfattah** (2016) introduced a new four parameters model called the Weibull Generalized Flexible Weibull extension [2]. **Zubair** (2017). Proposed a new model. This model is called new extended flexible Weibull distribution [21]. **Zubair** (2017). introduced a four-parameter modification of new flexible Weibull distribution [20]. **El-Desouky** (2017). devoted to study a new generalization of the flexible Weibull with three parameters. This model is referred to as the exponential flexible Weibull extension [5]. **Khaleel** (2020). Introduced the Gompertz flexible Weibull distribution as an extension of the flexible Weibull distribution [10]. **NihadSh** (2022) developed The Topp Leone Flexible Weibull distribution: An extension of the Flexible Weibull distribution [14].

2- **transformation the log-expo transformation (LET): LET-FW.**

Aslam et al. (2020) [4] developed a generator to propose new continuous lifetime distributions defined as:

Let F (x; z) be the cdf of a given random variable depending on some real-valued parameter (s) z. Our approach in this paper consists in enriching this cdf by transforming it into:

$$G(X;\xi)=\frac{\log(2-e^{-\lambda F(x)})}{\log(2-e^{-\lambda})}.....(1)$$

where ξ = (λ, ζ) for some positive real-valued shape parameter λ and the parameter ζ from the baseline distribution.We call this transformation the log-expo transformation (LET).

The probability density function (pdf) corresponding to Eq (1) is given by

$$\text{g}(X;\xi)=\frac{\lambda f(x;\nu)e^{-\lambda F(x;\nu)}}{\log(2-e^{-\lambda})(2-e^{-\lambda F(x;\nu)})}\ \text{x}>0.....(2)$$

f(.) and F(.) are the PDF and the CDF of flexible weibull distribution, where:

$$f(x)=(\alpha+\tfrac{\beta}{x^2})\exp(\alpha\,\text{x}-\tfrac{\beta}{x})\exp(-\text{e}^{\alpha\text{x}-\frac{\beta}{x}}),\alpha,\beta,x>0...(3)$$

$$F(x)=1-\exp(-\text{e}^{\alpha\text{x}-\frac{\beta}{x}})...(4)$$

We obtain the PDF and the CDF of LET-flexible weibull (LET-FW) by putting (3) and (4) in (1) and (2) as:

$$g(x) = \frac{\lambda z(\alpha + \frac{\beta}{x^2})e^{-[z+\lambda(1-e^{-z})]}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-e^{-z}]})} \;.....(5)$$

$$where: z = \exp(\alpha x - \tfrac{\beta}{x})$$

$$\lambda, \alpha, \beta > 0, x > 0$$

$$G(x) = \frac{\log(2-e^{-\lambda[1-e^{-z}]})}{\log(2-e^{-\lambda})} \;...(6)$$

$$where: z = \exp(\alpha x - \tfrac{\beta}{x})$$

$$\lambda, \alpha, \beta > 0, x > 0$$

Where $x = 0 \rightarrow G(0) = 0 \;\&\; x = \infty \rightarrow G(\infty) = 1$

By the PDF (5) and the CDF (6) we can write the survival function s(x) and the hazard function h(x) as:

$$s(x) = 1 - G(x)$$

$$\therefore s(x) = 1 - \frac{\log(2-e^{-\lambda[1-e^{-z}]})}{\log(2-e^{-\lambda})}$$

$$where: z = \exp(\alpha x - \tfrac{\beta}{x})$$

$$h(x) = \frac{g(x)}{s(x)}$$

$$h(x) = \frac{\lambda z\,(\alpha + \frac{\beta}{x^2})e^{-[z+\lambda(1-e^{-z})]}}{\log[(2-e^{-\lambda})/(2-e^{-\lambda[1-e^{-z}]})](2-e^{-\lambda[1-e^{-z}]})}$$

$$where : z = \exp(\alpha x - \tfrac{\beta}{x})$$

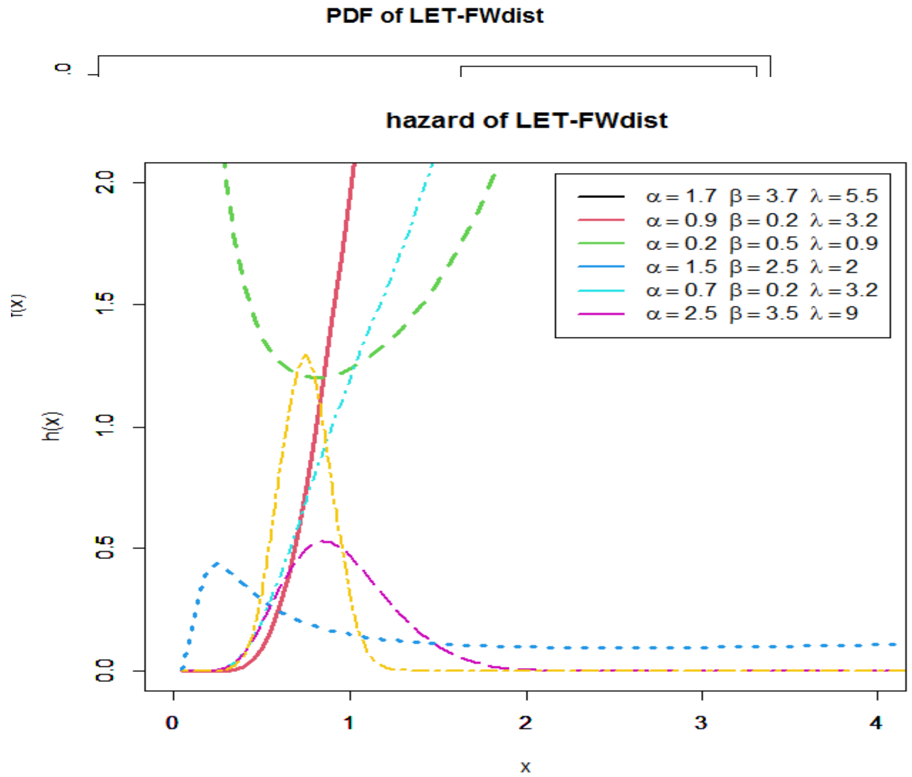


**Figure 2. Hazard of LET-FW distribution**

**3-Quantile function**

The Quantile function Q(p) or Q of the LET-FW distribution is the solution of the equation: G(Q)=P. then the quantile function Q at a vector p of percentiles is:

$$Q = \frac{t + \sqrt{t^2 - 4\alpha\beta}}{2\alpha} \quad ........(7)$$

Where $t = \log\{-\log[1+\frac{\log(2-\exp(P\log(2-e^{-\lambda})))}{\lambda}]$

We can obtain the generating function of the LET-FW distribution from equation (7) by writing $x_u$ instead of Q and u instead of p, Where u is the uniform random variable (0,1) then:

$$\therefore x_u = \frac{t+\sqrt{t^2-4\alpha\beta}}{2\alpha}, \alpha,\beta,\lambda \succ 0$$

Where $t = \log\{-\log[1+\frac{\log(2-\exp(u\log(2-e^{-\lambda})))}{\lambda}]$

The Bowley Skewness measure Bsk [9] and the Moor's Kurtosis measure Mkur [13] are defined by

$$Bsk = \frac{Q_{0.75}-2Q_{0.5}+Q_{0.25}}{Q_{0.75}-Q_{0.25}}$$

$$Mkur = \frac{Q_{0.875}-2Q_{0.625}-2Q_{0.375}+Q_{0.125}}{Q_{0.75}-Q_{0.25}}$$

Then the skewness and kurtosis of LET-FW for different parameter values are given in the following table.

**table. 1 Quantile**

| $\lambda,\alpha,\beta$ | $Q_{0.25}$ | $Q_{0.5}$ | $Q_{0.75}$ | $Q_{0.125}$ | $Q_{0.375}$ | $Q_{0.625}$ | $Q_{0.875}$ | Bsk | Mkur |
|---|---|---|---|---|---|---|---|---|---|
| **0.5,0.7,0.9** | 0.2192 | 0.3255 | 0.4799 | 0.1679 | 0.2696 | 0.3924 | 0.6121 | 0.1847 | 0.4525 |
| **0.9,1.2,1.5** | 0.3562 | 0.5193 | 0.7675 | 0.2773 | 0.4332 | 0.6247 | 0.9969 | 0.2069 | 0.5258 |
| **1.2,1.5,0.9** | 0.5261 | 0.7811 | 1.1516 | 0.4029 | 0.6470 | 0.9418 | 1.4690 | 0.1847 | 0.4525 |
| **1.5,1.2,2** | 0.5546 | 0.7958 | 1.1678 | 0.4368 | 0.6687 | 0.9522 | 1.5273 | 0.2134 | 0.5597 |
| **1.5,2.5,2** | 0.5546 | 0.7958 | 1.1678 | 0.4368 | 0.6687 | 0.9522 | 1.5273 | 0.2134 | 0.5597 |
| **1.7,3.7,5.5** | 0.4811 | 0.6389 | 0.8628 | 0.3965 | 0.5578 | 0.7347 | 1.0786 | 0.1728 | 0.4785 |
| **0.9,0.2,3.2** | 0.2945 | 0.4084 | 0.5814 | 0.2368 | 0.3488 | 0.4809 | 0.7553 | 0.2060 | 0.5659 |

From Table 1. drawn above it can be deduced that the LET-FW distribution can be used to model data that are positively skewed.

**4-Raw Moments**

the r-th raw moment of the LET-FW variable x is

$$\mu_r' = \mathrm{E}(\mathrm{x}^r)$$

$$= \int_a^b x^r \frac{\lambda z\,(\alpha + \frac{\beta}{x^2})\exp\{-z - \lambda(1-e^{-z})\}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-\mathrm{e}^{-z}]})} dx$$

$$where: z = \exp\{\alpha x + \tfrac{\beta}{x}\}$$

The mean of x corresponds to r = 1. The mean, variance, skewness and kurtosis of the distribution for various values of the parameters are shown in Table 1. Table 1 indicates that if a, b and λ are fixed, the mean and variance of the LET-FW distribution

| $\lambda,\alpha,\beta$ | $\mu_1'$ | $\mu_2'$ | $\mu_3'$ | $\mu_4'$ | $\mu_2$ | CV | sk | kur |
|---|---|---|---|---|---|---|---|---|
| **0.5,0.7,0.9,4** | 0.1813 | 0.2535 | 0.4964 | 1.1882 | 0.2206 | 2.5901 | 3.5760 | 14.9794 |
| **0.9,1.2,1.5,4** | 0.2566 | 0.2551 | 0.3111 | 0.4441 | 0.1893 | 1.6954 | 1.8032 | 2.9315 |
| **1.2,1.5,0.9,4** | 0.1868 | 0.1883 | 0.2160 | 0.2747 | 0.1534 | 2.0971 | 2.0552 | 3.3333 |
| **1.5,1.2,2,4** | 0.2322 | 0.1645 | 0.1353 | 0.1261 | 0.1106 | 1.4324 | 1.2422 | 0.6708 |

**5-Renyi Entropy:**

Entropy has been appeared by Alfred Renyi [3] as a logarithmic measure of variation of uncertainty.

If we assume that the events X=$\{x_1, x_2, \ldots\ldots \mathrm{x}_N\}$ have different probability $\{\mathrm{p}_1, \mathrm{p}_2, \ldots\ldots \mathrm{p}_N\}$. And each deliver $\mathrm{I}_k$ bits of information, then the total amount of information for the set is $\mathrm{I}_1(\mathrm{p}) = \sum_{k=1}^{N} p_k I_k$

Appling the definition to the I(p) we got $\mathrm{I(p)} = g^{-1}(\sum_{k=1}^{N} p_k\, g(I_k))$

When the postulate of additively for independent events is applied, we get just two possible g(x):

g(x)=cx

$$\mathrm{g(x)=c}^{-2(1-\alpha)\mathrm{x}}$$

The first form gives Shannon information and the second gives

$$\mathrm{I}_\alpha(\mathrm{p}) = \tfrac{1}{1-\alpha}\log \sum_{k=1}^{N} p_k^\alpha$$

In continuous distribution $I_\alpha(p) = \frac{1}{1-\alpha}\log\int_0^\infty g^p(x)dx, \alpha > 0, \alpha \neq 0$ i

Hence,

$$I_\alpha(p)$$

$$= \frac{1}{1-\alpha}\log\int_0^\infty \{\frac{\lambda z(\alpha+\frac{\beta}{x^2})\exp\{-z-\lambda(1-e^{-z})\}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-e^{-z}]})}\}^p dx$$

$$where: z = \exp\{\alpha x + \frac{\beta}{x}\}$$

The following Table below gives the values of Renyi Entropy of LET-FW distribution for different values of the parameters.

**table .3 Renyi Entropy**

| $\lambda,\alpha,\beta$ | **p=2** | **p=3** | **p=4** |
|---|---|---|---|
| **0.9,0.7,0.5** | 3.387055 | 2.465451 | 2.119736 |
| **0.9,1.2,1.5** | 3.246222 | 2.415173 | 2.118701 |
| **1.5,0.9,1.2** | 2.397793 | 1.750885 | 1.509868 |
| **2, 1.5, 2.5** | 1.9248 | 1.381453 | 1.180555 |
| **2, 1.5, 1.2** | 1.689384 | 1.135174 | 0.928305 |
| **5.5,1.7, 3.7** | 1.250535 | 0.808847 | 0.64208 |
| **3.2, 0.9, 0.2,** | 0.105591 | -0.46075 | -0.68114 |
| **4.5, 0.7, 0.4** | -0.31782 | -0.31782 | -0.31782 |

It should be noted that the higher the value of the Renyi entropy the greater the level of uncertainty in the system.

**6-The mean Residual life:**

The mean Residual life (MRL) or the life expectancy at age t is the expected additional life length for a unit, which is alive at age t. the MRL is given by:

$$m(t) = (\frac{1}{1-F(x)}\int_t^\infty xg(x)dx) - t$$

$$= (\frac{1}{\exp\{-z-\lambda(1-e^{-z})}\int_t^\infty \frac{\lambda z(\alpha x+\frac{\beta}{x})\exp\{-z-\lambda(1-e^{-z})\}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-e^{-z}]})}dx) - t$$

$$where, z = e^{\alpha x - \frac{\beta}{x}}$$

The following Table below gives the values of MRL of LET-FW distribution for a fixed value of t.

**TABLE.4 Mean residual life, α=0.5, β =0.7, λ =0.9**

| NO. | Death. Time | EMRL | NO. | Death.Time | EMRL |
|---|---|---|---|---|---|
| 1 | 0.091065 | 0.256272 | 11 | 0.212294 | 0.177682 |
| 2 | 0.12218 | 0.229848 | 12 | 0.216801 | 0.177732 |
| 3 | 0.126223 | 0.230609 | 13 | 0.222688 | 0.176489 |
| 4 | 0.162771 | 0.19828 | 14 | 0.225212 | 0.178798 |
| 5 | 0.182556 | 0.182462 | 15 | 0.225859 | 0.183241 |
| 6 | 0.191661 | 0.177296 | 16 | 0.232166 | 0.182138 |
| 7 | 0.193951 | 0.179076 | 17 | 0.233552 | 0.186229 |
| 8 | 0.199815 | 0.177336 | 18 | 0.235976 | 0.189549 |
| 9 | 0.206831 | 0.174474 | 19 | 0.240115 | 0.191391 |
| 10 | 0.212165 | 0.173369 | 20 | 0.240906 | 0.196954 |

EMRL decreases with increasing death time

## 7-Methods of Estimation

In this section we estimate the parameters of the LET-FW distribution by 5 different methods using complete sample technique. These methods are: Maximum likelihood (MLE), Least-squares (LS), weighted least squares, Maximum Product of spacing (MPS) and percentile-based estimation (PE). The performance of all methods is studied by the R software.

### 7.1-Maximum likelihood estimation (MLE):

The MLE method is a general method, and its estimators have some optimum properties such as consistency, asymptotic efficiency and invariance property.

Let $x_1, x_2, \ldots x_n$ be a random sample from LET-FW population with PDF g(x) given in (5) with unknown parameters α, β, and λ and the log-likelihood function is l (α, β, λ) then the MLE estimates of α, β, and λ are the simultaneously solution of the following equations:

$$\frac{\partial \ell(\alpha,\ \beta,\ \lambda,\theta)}{\partial \alpha}=0,\ \frac{\partial \ell(\alpha,\ \beta,\ \lambda,\theta)}{\partial \beta}=0,\ \frac{\partial \ell(\alpha,\ \beta,\ \lambda,\theta)}{\partial \lambda}=0$$

We derive sample estimates of the unknown parameters of the LET model by using the maximum likelihood estimation technique. Let $x_1, x_2, \ldots\ldots x_n$ be the observations of a random sample of size n from the LET model. The likelihood function is given by [4]

$$L(\Omega)=\prod_{i=1}^{n}\frac{\lambda f\,(x;\Omega)\exp\{-\lambda F(x;\Omega)\}}{\log[2-e^{-\lambda}][2-\exp\{-\lambda F(x;\Omega)\}]}$$

and the log-likelihood function by

$$\ell\ell(\Omega)=n\log\lambda+\sum\log\{f\,(x;\Omega)\}-\lambda\sum F(x;\Omega)-n\log[\log(2-e^{-\lambda})]$$
$$-\sum\log[2-\mathrm{e}^{-\lambda F(x;\Omega)}]\text{.................................}(8)$$

For obtaining the partial derivatives, differentiating (8) for and we get,

$$\frac{\partial\ell\ell(\Omega)}{\partial\lambda}=\frac{n}{\lambda}-\sum 1-e^{-e^{\alpha x-\beta x^{-1}}}-\frac{ne^{-\lambda}}{(2-e^{-\lambda})\log(2-e^{-\lambda})}$$
$$-\sum\frac{(1-e^{-e^{\alpha x-\beta x^{-1}}})\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}{2-\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}=0\text{............}(9)$$

$$\frac{\partial\ell\ell(\Omega)}{\partial\alpha}=\sum\frac{x^{2}(1-e^{-e^{\alpha x-\beta x^{-1}}})}{\alpha}(\alpha+\beta x^{-2})^{-1}-\lambda\sum xe^{\alpha x-\beta x^{-1}}e^{-e^{\alpha x-\beta x^{-1}}}$$
$$-\sum\frac{xe^{\alpha x-\beta x^{-1}}e^{-e^{\alpha x-\beta x^{-1}}}\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}{2-\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}=0\text{............}(10)$$

$$\frac{\partial\ell\ell(\Omega)}{\partial\beta}=\sum x\,(1-e^{\alpha x-\beta x^{-1}})+\lambda\sum x^{-1}e^{\alpha x-\beta x^{-1}}e^{-e^{\alpha x-\beta x^{-1}}}$$
$$+\sum\frac{x^{-1}e^{\alpha x-\beta x^{-1}}e^{-e^{\alpha x-\beta x^{-1}}}\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}{2-\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\}}=0\text{............}(11)$$

Setting (9), (10) and (11) to zero and solving these equations simultaneously gives the MLE of and i.e., and. However, solving these equations to get the estimates of the unknown parameter is quite difficult. Therefore, a numerical technique such as the newton-raphson method may be used to solve these non-linear equations.

**7.2-Method of Ordinary Least Squares (L):**

The best estimates according to LS method are those which minimize the following quantity:

$$Q_1=\sum_{i=1}^{n}(G(x_{(i)}-\frac{i}{n+1}))^2$$

With respect to α, β and λ.

Where $x_{(i)}$ is the I th orders statistic of LET-FW

Similarly, these estimators are also obtained by solving the following equation (for k = 1 ,2 ,3 ,4) (see [6])

$$Q=[\frac{\log(2-\exp\{-\lambda[1-e^{-e^{\alpha x-\beta x^{-1}}}]\})}{\log(2-e^{-\lambda})}-\frac{i}{n+1}]\Psi_k(x_{(i)}/\Omega)=0$$

Where

$$\Psi_1(x_{(i)}/\Omega)=\frac{\partial G(x)}{\partial\lambda}=\frac{\lambda e^{-\lambda}\{[\log(2-e^{-\lambda})]^2 e^{(1-w)}+[\log(2-e^{-\lambda(1-w)})]^2\}}{\log(2-e^{-\lambda})\log(2-e^{-\lambda(1-w)})}$$

$$\Psi_2(x_{(i)}/\Omega)=\frac{\partial G(x)}{\partial\alpha}=\frac{-(\lambda/x)w(\log w)e^{-\lambda(1-w)}}{\log(2-e^{-\lambda(1-w)})}$$

$$\Psi_3(x_{(i)}/\Omega)=\frac{\partial G(x)}{\partial\beta}=\frac{\lambda x w(\log w)e^{-\lambda(1-w)}}{\log(2-e^{-\lambda(1-w)})}$$

where $w=\exp\{-e^{\alpha x-\beta x^{-1}}\}$ .The solution of $\Psi_k$ for k = 1,2,3,4 may be obtained numerically.

**7.3-Method of Weighted Least Squares (WLS):**

The WLS estimators of α, β, and λ of LET-FW distribution can be obtained by minimizing the quantity:

$$Q_2=\sum_{i=1}^{n}\frac{(\text{n}+1)^2(n+2)}{n-i+1}(G(x_{(i)}-\frac{i}{n+1}))^2$$

With respect to α., β, and λ

**7.4-Method of Percentile Estimation (PCE) :**

This method introduced by kao [7, 8] the PCEs estimators of α, β, and λ of LET-FW distribution can be obtained by minimizing the quantity:

$$Q=\sum_{i=1}^{n}[x_{(i)}-\frac{t+\sqrt{t^2-4\alpha\beta}}{2\alpha}]^2$$

Where $t=\log\{-\log[1+\frac{\log(2-\exp(P\log(2-e^{-\lambda})))}{\lambda}]$

**7.5-Simulation Study and Data Analysis**

The aim of this section is to compare the performance of the methods of estimation, namely: MLE, MPS, LS, WLS, and PE for the LET-FW distribution which discussed in the previous section. A Monte Carlo study is employed to check the behavior of the proposed methods of estimation. Also, a real data set is analyzed for illustrative purpose. R-statistical programming language will be used for calculation.

**7.5.1-Simulation Study**

A simulation study is employed to compare the performance of proposed methods of estimation using Monte Carlo. The Monte Carlo process is carried by generating 5000 random data from the LET-FW distribution with the following assumptions:

1. Sample sizes are $n = 50, 100, 150, 200$
2. Assume the following selected cases of parameters α and β of the LET-FW 3rd distribution:
    a. $\alpha = 1.2, \beta = 2, \lambda = 4$
    b. $\alpha = 0.5, \beta = 0.7, \lambda = 0.9$

Based on the generated data and applying different methods of estimation. All the means square error (MSE) and relative biases (BIAS) are reported from Table (5) to Table (8) for five different methods of estimation.

**Table (5): The MSE and BIAS for different estimates of the LET-FW distribution with n= 200**

| | MLE | | MPS | | LSE | | WLSE | | PE | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Parameters** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** |
| **case \|** | | | | | | | | | | |
| **α = 1.2** | **0.616496** | 0.64932 | **0.13364** | 0.186632 | **0.480028** | 0.360602 | **0.343588** | 0.348281 | **0.012991** | 0.00286 |
| **β = 2** | **0.434049** | 0.327603 | **0.671499** | 0.405988 | **0.662265** | 0.397548 | **0.647193** | 0.395986 | **1.990717** | 0.257411 |
| **λ = 4** | **8.380159** | 0.690051 | **2.759733** | 0.059395 | **14.45506** | 0.245171 | **7.004066** | 0.157858 | **1.007737** | 0.005931 |
| **case \|\|** | | | | | | | | | | |
| **α = 0.5** | **0.122319** | 0.684559 | **2.517458** | 3.154277 | **2.201844** | 2.911553 | **2.243438** | 2.965425 | **0.003342** | 0.037965 |
| **β = 0.7** | **0.003187** | 0.052026 | **0.040197** | 0.274817 | **0.046604** | 0.289153 | **0.044598** | 0.287616 | **0.252265** | 0.108033 |
| **λ = 0.9** | **16.33434** | 4.477 | **0.217618** | 0.01616 | **0.534472** | 0.096433 | **0.323855** | 0.054066 | **0.409235** | 0.214932 |

**Table (6): The MSE and BIAS for different estimates of the LET-FW distribution with n= 150**

| **Parameters** | MLE | | MPS | | LSE | | WLSE | | PE | |
|---|---|---|---|---|---|---|---|---|---|---|
| **.** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** |
| **case \|** | | | | | | | | | | |
| **α = 1.2** | **0.619616** | 0.649224 | **0.152874** | 0.163355 | **0.540322** | 0.367439 | **0.394422** | 0.359114 | **0.015762** | 0.000207 |
| **β = 2** | **0.432702** | 0.326477 | **0.677594** | 0.406638 | **0.676602** | 0.399485 | **0.653944** | 0.396233 | **2.936521** | 0.285161 |
| **λ = 4** | **8.904842** | 0.70012 | **3.795244** | 0.061588 | **19.09814** | 0.288819 | **9.447946** | 0.195857 | **1.26137** | 0.016602 |
| **case \|\|** | | | | | | | | | | |
| **α = 0.5** | **0.123488** | 0.682512 | **2.574445** | 3.183128 | **2.171876** | 2.859347 | **2.218424** | 2.933811 | **0.005248** | 0.053862 |
| **β = 0.7** | **0.003971** | 0.053644 | **0.03994** | 0.26991 | **0.049121** | 0.28997 | **0.04615** | 0.287522 | **0.407575** | 0.087416 |
| **λ = 0.9** | **16.52208** | 4.49738 | **0.291019** | 0.038525 | **0.873238** | 0.155786 | **0.489165** | 0.091305 | **0.637385** | 0.303331 |

**Table (7): The MSE and BIAS for different estimates of the LET-FW distribution with n= 100**

| Parameters | MLE | | MPS | | LSE | | WLSE | | PE | |
|---|---|---|---|---|---|---|---|---|---|---|
| . | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias |
| case \| | | | | | | | | | | |
| α = 1.2 | **0.62717** | 0.648928 | **0.208926** | 0.129269 | **0.678321** | 0.382291 | **0.503767** | 0.37089 | **0.020819** | 0.001967 |
| β = 2 | **0.427403** | 0.322943 | **0.684219** | 0.405584 | **0.704576** | 0.402034 | **0.673211** | 0.397538 | **2.368339** | 0.306453 |
| λ = 4 | **10.674** | 0.731419 | **7.075583** | 0.035797 | **36.70636** | 0.42247 | **19.69992** | 0.295698 | **1.754424** | 0.017292 |
| case \|\| | | | | | | | | | | |
| α = 0.5 | **0.131423** | 0.693565 | **2.729861** | 3.264185 | **2.166979** | 2.787758 | **2.216589** | 2.894623 | **0.007711** | 0.068609 |
| β = 0.7 | **0.005658** | 0.059692 | **0.040613** | 0.263337 | **0.054306** | 0.290954 | **0.050294** | 0.289081 | **0.509271** | 0.065401 |
| λ = 0.9 | **16.88202** | 4.532799 | **0.495875** | 0.050664 | **1.983776** | 0.268424 | **1.096127** | 0.152403 | **0.923971** | 0.372343 |

**Table (8): The MSE and BIAS for different estimates of the LET-FW distribution with n= 50**

| Parameters | MLE | | MPS | | LSE | | WLSE | | PE | |
|---|---|---|---|---|---|---|---|---|---|---|
| . | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias |
| case \| | | | | | | | | | | |
| α = 1.2 | **0.637864** | 0.642906 | **0.387293** | 0.040868 | **0.983082** | 0.412819 | **0.787634** | 0.406681 | **0.033822** | 0.006683 |
| β = 2 | **0.420285** | 0.315911 | **0.698617** | 0.401638 | **0.782257** | 0.410949 | **0.732183** | 0.402658 | **3.026804** | 0.337227 |
| λ = 4 | **16.75231** | 0.815134 | **20.44738** | 0.062346 | **94.63256** | 0.717014 | **60.02438** | 0.575011 | **3.236457** | 0.036112 |
| case \|\| | | | | | | | | | | |
| α = 0.5 | **0.150411** | 0.711585 | **3.154388** | 3.466396 | **2.1749** | 2.595215 | **2.173154** | 2.738738 | **0.011845** | 0.089169 |
| β = 0.7 | **0.010357** | 0.070355 | **0.04145** | 0.243577 | **0.067525** | 0.294601 | **0.060576** | 0.290315 | **0.758217** | 0.029168 |
| λ = 0.9 | **17.89873** | 4.629399 | **1.049375** | 0.11494 | **4.712268** | 0.556691 | **2.871747** | 0.376766 | **1.236915** | 0.443736 |

**7.5.2 -Data Analysis**

From the above tabulated results, one can indicate that:

The MSEs decrease as sample size increases.

Comparing the different methods of estimation, the results show that the MLE produces the best results for estimating the parameter β and the PE produces the best results for parameters (α, λ).

The ordering performance of the estimators in terms of LSs (from best to worst) for α PE and MPS. For β is MLE and PE. While the ordering performance for λ is PE and MLE.

**8-Censored samples and types of censoring**

In life testing experiments, items drawn from a population are put to test and their times to failure are noted. The test is usually terminated after a fixed time or after a fixed number of failures is observed, giving a censored sample.

The situation in which we observe the lifetimes of all the items is called uncensored data, but such data will rarely arise in reliability testing even under controlled conditions. It is often necessary to terminate the test before all failure items have been observed because of limited budget or time.

An observation is said to be right censored at L in the exact value of the observation is not known but it is known only that it is greater than or equal to L . similarly, an observation is said to be left censored at M if it is known only that the observation is less than or equal to M. right censoring is very common in lifetime data, but left censoring is fairly rare.

Some methods by which the data may be censored are:

**Type I censored data**

Suppose that a random sample of n units from $G(x,\Omega)$ is processed for a predefined time $x_c$ and then the process terminates. We observed the lifetime of $\delta$ observations before terminating the process and the remaining $n-\delta$ observations will be censored. Thus, the lifetimes are observed only if $x_i < x_c$ for $i = 1,2,.....n$ .

$$Defining I_i \begin{cases} 1, if x_i \le x_c \\ 0, if x_i > x_c \end{cases} and\, \delta = \sum_{i=1}^{n} I_i$$

the likelihood function can be written as

$$L(\Omega)=[\prod_{i=1}^{n} g(x;\Omega)^{I_i}]S(x;\Omega)^{n-\delta} \quad [4]$$

and the log-likelihood function is given by

$$\ell(\Omega)=(n-\delta)\log[1-\frac{\log(2-e^{-\lambda F(x_c;\Omega)})}{\log(2-e^{-\lambda})}]+\sum_{i=1}^{n} I_i \log[\frac{\lambda f(x;\Omega)e^{-\lambda F(x_c;\Omega)}}{\log(2-e^{-\lambda})(2-e^{-\lambda F(x_c;\Omega)})}]$$

then

$$\ell\ell(\Omega)$$

$$=(n-\delta)\log[1-\frac{\log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_c-\beta x_c^{-1}}\}]})}{\log(2-e^{-\lambda})}]+\sum_{i=1}^{n} I_i \log[\frac{\lambda(\alpha+\beta x^{-2})e^{\alpha x_c-\beta x_c^{-1}}\exp\{-e^{\alpha x_c-\beta x_c^{-1}}\}e^{-\lambda[1-\exp\{-e^{\alpha x_c-\beta x_c^{-1}}\}]}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-\exp\{-e^{\alpha x_c-\beta x_c^{-1}}\}]})}]$$

**Type II censored data**

In case of Type-II right censoring, t observations out of the n are censored from the right side. The likelihood function then becomes

$$L(\Omega)=\frac{n!}{t!}[\prod_{i=1}^{n-t} g(x_i;\Omega)][S(x_{n-t};\Omega)]^t$$

where $x_i$ is the order statistic of order i, and the log-likelihood function, expressed in terms of the original baseline distribution, reads

$$\ell(\Omega)=\log(\frac{n!}{t!})+t\log[1-\frac{\log(2-e^{-\lambda F(x_{n-t};\Omega)})}{\log(2-e^{-\lambda})}]+\sum_{i=1}^{n-t} I_i \log[\frac{\lambda f(x_{(i)};\Omega)e^{-\lambda F(x_{(i)};\Omega)}}{\log(2-e^{-\lambda})(2-e^{-\lambda F(x_{(i)};\Omega)})}]$$

$$\ell(\Omega)=\log(\frac{n!}{t!})+t\log[1-\frac{\log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})}{\log(2-e^{-\lambda})}]$$

$$+\sum_{i=1}^{n-t} I_i \log[\frac{\lambda(\alpha+\beta x_{(i)}^{-2})e^{\alpha x_{(i)}-\beta x_{(i)}^{-1}}\exp\{-e^{\alpha x_{(i)}-\beta x_{(i)}^{-1}}\}e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}{\log(2-e^{-\lambda})(2-e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]})}]=0$$

$$\frac{\partial\ell\ell(\Omega)}{\partial\lambda}=\frac{n-t}{\lambda}-k_1(k_2-k_3)$$

$$-\sum_{i=1}^{n-t}[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}+\frac{e^{-\lambda}}{(2-e^{-\lambda})\log(2-e^{-\lambda})}+\frac{(1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\})e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}{2-e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}]=0$$

Where:

$$k_1 = \frac{t}{\log(2-e^{-\lambda})[\log(2-e^{-\lambda}) - \log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})}$$

$$k_2 = \frac{\log(2-e^{-\lambda})[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]}}{2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]}}$$

$$k_3 = \frac{\log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})e^{-\lambda}}{2-e^{-\lambda}}$$

$$\frac{\partial \ell\ell(\Omega)}{\partial \alpha} = \frac{-t\lambda x_{n-t} e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}} e^{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}} e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]}}{(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})[\log(2-e^{-\lambda})-\log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})]} + \sum_{i=1}^{n-t}[\frac{x_i^2(1-e^{\alpha x_i-\beta x_i^{-1}})}{\alpha} + (\alpha+\beta x_i^{-2})^{-1}]$$

$$-\lambda\sum_{i=1}^{n-t}[\frac{x e^{\alpha x_i-\beta x_i^{-1}} e^{-e^{\alpha x_i-\beta x_i^{-1}}}}{1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}}] - \lambda\sum_{i=1}^{n-t}[\frac{x_i e^{\alpha x_i-\beta x_i^{-1}} e^{-e^{\alpha x_i-\beta x_i^{-1}}} e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}{2-e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}] = 0$$

$$\frac{\partial \ell\ell(\Omega)}{\partial \beta} = \frac{-t\lambda x_{n-t}^{-1} e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}} e^{-e^{\alpha x-\beta x^{-1}}} e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]}}{(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})[\log(2-e^{-\lambda})-\log(2-e^{-\lambda[1-\exp\{-e^{\alpha x_{n-t}-\beta x_{n-t}^{-1}}\}]})]} + \sum_{i=1}^{n-t}[x(1-e^{\alpha x-\beta x^{-1}})]$$

$$-\lambda\sum_{i=1}^{n-t}[\frac{x_i^{-1} e^{\alpha x_i-\beta x_i^{-1}} e^{-e^{\alpha x_i-\beta x_i^{-1}}}}{1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}}] - \lambda\sum_{i=1}^{n-t}[\frac{x_i^{-1} e^{\alpha x_i-\beta x_i^{-1}} e^{-e^{\alpha x_i-\beta x_i^{-1}}} e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}{2-e^{-\lambda[1-\exp\{-e^{\alpha x_i-\beta x_i^{-1}}\}]}}] = 0$$

**Table.9. MLE Estimation under type-I censoring scheme n=25**

| Name | α=0.5 | β=0.7 | λ=0.9 | α=0.5 | β=0.7 | λ=0.9 |
|---|---|---|---|---|---|---|
| **Time of censoring (% 30)** | | | | **Time of censoring (% 60)** | | |
| **Bias** | 18.66094 | 0.368929 | 7.310087 | 5.469342 | 0.201593 | 6.788221 |
| **MSE** | 183.9349 | 0.217024 | 49.65025 | 19.45356 | 0.108474 | 43.32669 |
| **AIL** | 38.59205 | 1.520234 | 9.895382 | 13.56835 | 1.166825 | 9.606604 |
| **CP** | 100 | 97.25 | 94.38776 | 100 | 94.1 | 94.14536 |
| **Time of censoring (% 90)** | | | | **Time of censoring (% 99.99)** | | |
| **Bias** | 0.502595 | 0.106925 | 5.684782 | 0.736369 | 0.09749 | 4.959587 |
| **MSE** | 0.781198 | 0.034759 | 30.33054 | 0.214299 | 0.023833 | 21.80851 |
| **AIL** | 3.322486 | 0.669505 | 7.99143 | 1.100228 | 0.542951 | 5.382554 |
| **CP** | 99.6 | 96.15 | 94.93136 | 97.1 | 96.6 | 95.98394 |

**Table.10. MLE Estimation under type-I censoring scheme (n=50)**

| Name | α=0.5 | β=0.7 | λ=0.9 | α=0.5 | β=0.7 | λ=0.9 |
|---|---|---|---|---|---|---|
| Time of censoring (% 30) | | | | Time of censoring (% 60) | | |
| Bias | 13.1507 | 0.167236 | 7.465081 | 3.53562 | 0.069969 | 6.768525 |
| MSE | 80.68157 | 0.083021 | 50.75216 | 6.292838 | 0.028787 | 42.42405 |
| AIL | 23.99338 | 1.032298 | 9.290721 | 6.978429 | 0.636934 | 9.040172 |
| CP | 99.9 | 93.1 | 93.42533 | 100 | 95.9 | 93.75355 |
| Time of censoring (% 90) | | | | Time of censoring (% 99.99) | | |
| Bias | 0.252295 | 0.060709 | 5.357527 | 0.697425 | 0.066563 | 4.617337 |
| MSE | 0.19266 | 0.010372 | 24.97674 | 0.146293 | 0.010034 | 17.78777 |
| AIL | 1.648401 | 0.362897 | 5.153031 | 0.616127 | 0.347676 | 2.823951 |
| CP | 98.7 | 96.45 | 96.42677 | 96.4 | 97.05 | 95.7 |

**Table.11. MLE Estimation under type-I censoring scheme (n=75)**

| Name | α=0.5 | β=0.7 | λ=0.9 | α=0.5 | β=0.7 | λ=0.9 |
|---|---|---|---|---|---|---|
| Time of censoring (% 30) | | | | Time of censoring (% 60) | | |
| Bias | 10.97393 | 0.103652 | 7.445455 | 2.941758 | 0.027718 | 6.533631 |
| MSE | 54.47567 | 0.050209 | 49.70957 | 3.337862 | 0.012275 | 37.99613 |
| AIL | 19.35551 | 0.831234 | 8.597914 | 4.249037 | 0.427686 | 7.249638 |
| CP | 99.95 | 93.55 | 94.44856 | 99.8 | 95.95 | 95.62433 |
| Time of censoring (% 90) | | | | Time of censoring (% 99.99) | | |
| Bias | 0.233561 | 0.047731 | 5.208587 | 0.698732 | 0.065513 | 4.591746 |
| MSE | 0.111507 | 0.006442 | 22.94296 | 0.136544 | 0.007128 | 17.3837 |
| AIL | 1.226618 | 0.286128 | 3.857996 | 0.471936 | 0.277946 | 2.16734 |
| CP | 98.65 | 96.7 | 96.94847 | 96.65 | 97.1 | 95.95 |

**Table.12. MLE Estimation under type-I censoring scheme n=100**

| Name | α=0.5 | β=0.7 | λ=0.9 | α=0.5 | β=0.7 | λ=0.9 |
|---|---|---|---|---|---|---|
| Time of censoring (% 30) | | | | Time of censoring (% 60) | | |
| Bias | 9.553252 | 0.053174 | 7.549929 | 2.740278 | 0.012746 | 6.557685 |
| MSE | 38.72291 | 0.030942 | 51.00367 | 2.478933 | 0.005986 | 38.08188 |
| AIL | 15.63787 | 0.674081 | 8.620099 | 3.041303 | 0.301327 | 7.067776 |
| CP | 100 | 93.6 | 93.7415 | 99.65 | 95.85 | 95.46392 |
| Time of censoring (% 90) | | | | Time of censoring (% 99.99) | | |
| Bias | 0.190914 | 0.046368 | 5.11259 | 0.692779 | 0.061839 | 4.536351 |
| MSE | 0.077407 | 0.00479 | 21.65912 | 0.130586 | 0.00584 | 16.89286 |
| AIL | 1.024661 | 0.239683 | 2.735885 | 0.40369 | 0.246946 | 1.856921 |
| CP | 98.1 | 96.7 | 96.49825 | 96.45 | 96.4 | 96.15 |

**Table.13. MLE Estimation under type-|| censoring scheme (n=25)**

| Name | α=1.2 | β=2.0 | λ=2.5 | α=1.2 | β=2.0 | λ=2.5 |
|---|---|---|---|---|---|---|
| Time of censoring (% 30) | | | | Time of censoring (% 60) | | |
| Bias | 1.848557 | 0.07224 | 2.875782 | 1.472882 | 0.148814 | 2.9256 |
| MSE | 15.96578 | 0.427464 | 107.627 | 5.341196 | 0.353655 | 120.6223 |
| AIL | 13.03079 | 2.500143 | 29.32829 | 5.838452 | 2.018687 | 32.1261 |
| CP | 97.8 | 95.3 | 93.96067 | 99.4 | 94.45 | 93.17102 |
| Time of censoring (% 90) | | | | Time of censoring (% 99.99) | | |
| Bias | 0.810813 | 0.224332 | 2.014976 | 0.664566 | 0.243463 | 1.587606 |
| MSE | 1.078529 | 0.295409 | 51.11129 | 0.675635 | 0.294768 | 22.48834 |
| AIL | 1.42371 | 1.202825 | 19.89089 | 0.781311 | 0.942143 | 10.18154 |
| CP | 98.8 | 96.75 | 95.8523 | 96.53465 | 97.0297 | 95.54455 |

**Table.14. MLE Estimation under type-|| censoring scheme (n=50)**

| **parm** | **α=1.2** | **β=2.0** | **λ=2.5** | **α=1.2** | **β=2.0** | **λ=2.5** |
|---|---|---|---|---|---|---|
| **Time of censoring (% 30)** | | | | **Time of censoring (% 60)** | | |
| **Bias** | 2.406791 | 0.151269 | 2.966659 | 1.356629 | 0.221778 | 2.878048 |
| **MSE** | 15.93513 | 0.321378 | 110.0101 | 3.562864 | 0.286987 | 106.917 |
| **AIL** | 10.80473 | 1.879785 | 29.0817 | 3.745704 | 1.177875 | 29.11774 |
| **CP** | 99.5 | 93.6 | 93.45037 | 99.55 | 94.95 | 94.04117 |
| **Time of censoring (% 90)** | | | | **Time of censoring (% 99.99)** | | |
| **Bias** | 0.847804 | 0.250531 | 1.796532 | 0.682449 | 0.258561 | 1.401501 |
| **MSE** | 1.093698 | 0.282949 | 29.23044 | 0.68484 | 0.294122 | 14.18583 |
| **AIL** | 0.949684 | 0.700151 | 11.80084 | 0.466909 | 0.640752 | 5.418167 |
| **CP** | 98.25 | 96.25 | 96.63655 | 96.45 | 96.55 | 95.7979 |

**Table.15. MLE Estimation under type-|| censoring scheme (n=75)**

| **parm** | **α=1.2** | **β=2.0** | **λ=2.5** | **α=1.2** | **β=2.0** | **λ=2.5** |
|---|---|---|---|---|---|---|
| **Time of censoring (% 30)** | | | | **Time of censoring (% 60)** | | |
| **Bias** | 2.376767 | 0.17592 | 3.034973 | 1.34692 | 0.238462 | 2.810155 |
| **MSE** | 14.38871 | 0.298831 | 111.929 | 3.247861 | 0.287813 | 95.76949 |
| **AIL** | 9.805498 | 1.640416 | 28.91083 | 3.125487 | 0.963277 | 26.71252 |
| **CP** | 99.8 | 94.1 | 93.86139 | 99.9 | 95.45 | 94.60318 |
| **Time of censoring (% 90)** | | | | **Time of censoring (% 99.99)** | | |
| **Bias** | 0.846121 | 0.26167 | 1.671403 | 0.685401 | 0.261656 | 1.346967 |
| **MSE** | 1.061074 | 0.292488 | 21.74019 | 0.684942 | 0.289962 | 12.317 |
| **AIL** | 0.680791 | 0.534784 | 8.11189 | 0.36077 | 0.497621 | 3.876557 |
| **CP** | 98.55 | 96.25 | 96.8 | 96.25 | 97.1 | 96.4 |

**Table.16. MLE Estimation under type-|| censoring scheme n=100**

| parm | α=1.2 | β=2.0 | λ=2.5 | α=1.2 | β=2.0 | λ=2.5 |
|---|---|---|---|---|---|---|
| Time of censoring (% 30) | | | | Time of censoring (% 60) | | |
| Bias | 2.36531 | 0.200833 | 3.127899 | 1.321521 | 0.248773 | 2.736617 |
| MSE | 13.13739 | 0.297286 | 116.4435 | 2.978365 | 0.288643 | 88.81589 |
| AIL | 8.838175 | 1.445693 | 29.15808 | 2.669453 | 0.794797 | 25.41343 |
| CP | 99.95 | 93.3 | 93.97208 | 99.85 | 95.5 | 94.27083 |
| Time of censoring (% 90) | | | | Time of censoring (% 99.99) | | |
| Bias | 0.854554 | 0.261174 | 1.670675 | 0.686349 | 0.264133 | 1.310204 |
| MSE | 1.077274 | 0.286352 | 20.94553 | 0.684174 | 0.291005 | 11.31658 |
| AIL | 0.628526 | 0.455637 | 7.336199 | 0.299279 | 0.428453 | 3.005628 |
| CP | 98.4 | 96.15 | 97 | 96.15385 | 97.12821 | 95.79487 |

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# تحويلة Log – expo لتوزيع وايبل المرن ذي المعالم الثلاث

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**ملخص البحث**

هناك طريقة جديدة لتوليد توزيعات الحياة تسمي بطريقة تحويلة Log – expo, هذه الدراسة تقدم توزيعا جديدا باستخدام هذه الطريقة هو توزيع وايبل المرن باستخدام هذه التحويلة LET-FW. وقد تمت دراسة بعض الخصائص الرياضية لهذا التوزيع كما تم ايجاد دالة كثافة الاحتمال ودالتي البقاء والخطر. وتم الحصول على العزوم العادية ودالة الوسيط ومتوسط البقاء علي قيد الحياة و Renyi entropy. تم تقدير معالم التوزيع باستخدام خمس طرق للتقدير كما تمك ايجاد أفضل تقدير عن طريق المقارنة بين نتائج هذه الطرق. وقد تمت المقارنة باستخدام المعيار MSE (متوسط مربعات الخطأ) عن طريق تجارب المحاكاة بالاعتماد على برنامج R باختيار أحجام عينات مختلفة وقيم معلومات أولية مختلفة. وأخيرا تقدير معالم التوزيع في حالات البيانات غير الكاملة والبيانات المراقبة وذلك بالنسبة للنوعين الأول والثاني للبيانات.

**المصطلحات الرئيسية للبحث:** وايبل المرن، دالة الوسيط، LET-FW، *Rényi entropy*