Bent-cable Regression Applied to the Daily Maximum Temperature in Cairo Through 2021

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Dr. Samar Ahmed Helmy Abdelghany and Dr. Samah Kamal Abdelaziz

Abstract

We use the so-called bent-cable model to describe the daily maximum temperature in Cairo, Egypt through 2021which exhibit a potentially sharp change in slope. The model comprises two linear segments, joined smoothly by a quadratic bend. The class of bent cables includes, as a limiting case, the popular piecewise-linear model (with a sharp kink), otherwise known as the broken stick. Associated with bent-cable regression is the estimation of the bend width parameter, through which the abruptness of the underlying transition may be assessed. We present worked the recorded maximum temperature in Cairo to demonstrate the regularity and irregularity of bent-cable regression encountered in finite-sample settings. We also presented the projections of the bent-cable for 2022 compared to those of accuweather and broken stick projections. Practical conditions on the design are given to ensure regularity of the full bent-cable estimation problem, if the underlying bend segment has non-zero width. Under such conditions, the least-squares estimators are shown (i) to be consistent, and (ii) to asymptotically follow a multivariate normal distribution. Hence the least-squares estimators are used to analysis the considered data.

Keywords

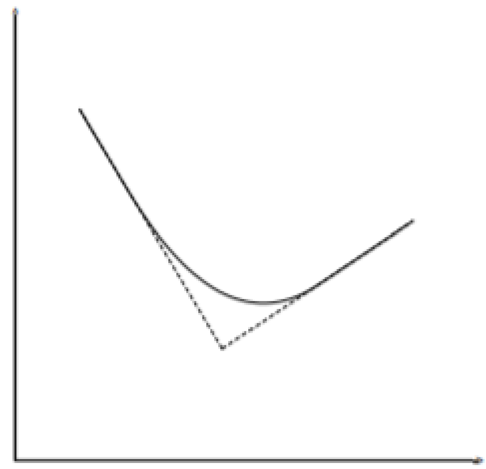
bent-cable - broken stick - change point - transition period - piecewise-linear model - maximum likelihood estimators - Köppen climate classification – Celsius.

1 -Introduction

The so-called broken stick is a popular piecewise-linear model used in meteorological, discovering inhomogeneities in temperature and biological studies for estimating the onset of change Barrowman and Myers (2000). This sharply kinked line is particularly appealing in its structural simplicity. For example, consider the relationship of abundance versus time for a declining fish population. A fisheries manager will naturally try to use the date of onset as a clue to the actual cause of the decline. In this and other fields, such as physiology, researchers are often tempted to conclude an abrupt onset from a broken-stick fit, although there is seldom solid theory to justify the abruptness Brown (1987).

To assess the nature of change, a non-parametric approach has been suggested in Routledge (1994), mostly as an exploratory tool when abruptness has not yet been verified. Now, we take a more formal parametric approach, by examining the use of a flexible model developed by Tishler and Zang, (1981). It is called the “bent-cable” due to the smooth bend as opposed to a sharp break in a snapped stick. The bent-cable can be regarded as an extension to the widely used broken stick with its simple structure retained. We exploit the fact that an extremely sharp bend reduces the bent-cable to a broken stick Figure (1) Bent-cable and broken stick models, Thus, besides relaxing the sometimes unreasonable a priori assumption of abruptness, the bent-cable also allows the data to reveal to us whether an abrupt change point or a smooth transition region is more convincing.

Figure (1)
Bent-cable and broken stick models



A bent-cable model (solid line) and a broken-stick model (dotted line). Beyond the edges of the middle quadratic segment of the bent-cable, both models have the same incoming and outgoing linear phases. Thus, the broken stick is a bent-cable whose smooth bend (transition region) is shrunk to a sharp break (change point). We also refer to the two-phase model as a bent-cable with zero or no bend (i.e., one with a missing middle segment).

The bent-cable model was originally developed to handle the non-differentiability of the first derivative when fitting a broken stick Tishler and Zang (1981). A "phoney" bend of fixed, non-trivial width replaces the kink. This was a computational tactic, as continuous differentiability provided more numerical stability in the estimation procedure. Upon numerical convergence, the phoney bend would be ignored. However, if a gradual transition is considered possible for the underlying regression function, then the bend width can be regarded as part of the parametric model.

However, formal assessments of abruptness via confidence regions rely ratio test procedures are inappropriate for a formal hypothesis test of an abrupt change versus a quadratic bend.

Fitting the bent-cable as an alternative model to the broken stick has been previously suggested by Seber and Wild (1989). However, neither they nor the inventors of the model provide estimation theory for it. The bent-cable methodology (Chiu et al. 2006) provides a regression framework to analyze change point data that exhibit a transition between two approximately linear phases. The model is parsimonious, and appealing due to its simple structure, great flexibility and interpretability.

(Chiu et al. 2006) presents the framework for the new bent-cable regression theory. Instead of considering multiple unknown parameters all at once, we confine our attention to the basic bent-cable — one with fixed slopes of 0 and 1, respectively, for the incoming and outgoing linear phases. The center and half-width of the bend are the only unknown parameters. Assuming normally distributed random errors with a known, constant variance, conditions on the design are given to ensure regularity of the estimation problem, despite non-differentiability of the model’s first partial derivatives (with respect to the covariate and model parameters). These practical conditions serve as a guideline for data collection in real-life settings.

Under such conditions, the maximum likelihood estimators (MLE’s) are shown:

(1) to be consistent, whether the underlying cable has a bend region or not; and

(2) asymptotically to follow a multivariate normal distribution, if the underlying cable has a non-trivial bend.

In the latter case, the deviance (or likelihood ratio test) statistic is shown to have an asymptotic chi-squared distribution with two degrees of freedom.

2 -The Basic Bent-Cable Regression Model

The regression function, evaluated at the covariate value x, is

$$f(x; \beta_0, \beta_1, \beta_2, \tau, \gamma) = \beta_0 + \beta_1 x + \beta_2 q(x; \tau, \gamma), \text{ where}$$

$$q(x;\tau,\gamma) = \frac{(x-\tau+\gamma)^2}{4\gamma} 1\{|x-\tau| \leq \gamma\} + (x-\tau) 1\{x-\tau > \gamma\} \quad (1)$$

is the so-called basic bent-cable

We consider the observations $\{(x_i , Y_i)\}$ i=1,…, n generated by a basic bent-cable. The regression model is

$Y_i = q(x_i ; \theta_0) + \varepsilon_i$, where ε_i are iid. ~ $N(0, \sigma^2)$ for a known σ^2 (2)

and q is the basic bent-cable, $\theta_0 = (\tau_0, \gamma_0)$ is the underlying bent-cable parameter.

y_i is the response at time t_i (i = 1, 2, . . . , n);

I(A) is an indicator function that equals 1 if A is true and otherwise;

β_0 and β_1 are the intercept and slope of the linear incoming phase respectively;

$\beta_1+\beta_2$ is the slope of the linear outgoing phase;

τ and γ are the transition parameters, characterizing the center and half-width of the bend, respectively; and

ε_i is the random error component.

For the estimation problem, we consider the unbounded parameter space, $\Omega = (-\infty, M] \times [0, \infty)$ for θ_0, and the open regression domain, X = R. Here, M is some large positive but finite upper bound for the candidate τ -values. The natural lower bound for candidate γ-values is, of course, zero. Any basic bent-cable q (x; θ) for θ ∈ Ω is a candidate model in estimating θ_0. While searching only within the class of basic bent-cables (as opposed to the class of all three-phase models in Feder (1975a), we do allow more flexibility than does Gallant in Gallant (1974) and Gallant (1975), or Ivanov (1997).

Besides an unspecified upper bound on the τ -space, our only regularity conditions are placed on the design points, $x_1, \ldots, x_n$. The practical value in these conditions is that they indicate a design which enables the investigator to ensure that data are collected at appropriate x-locations for reliable estimation of the underlying parameters.

3The Maximum Likelihood Estimator (MLE)

With normally distributed errors, $\widehat{\boldsymbol{\theta}}_{n}$, the MLE of θ_0, is equivalent to the least-squares estimator. In particular, the log-likelihood function for θ ∈ Ω is

$$l_n(\theta) = -\frac{1}{2\sigma^2}\sum_{i=1}\left[Y_i - q(x_i;\theta)\right]^2 + \text{ constant} \qquad (3)$$

which is linear in the error sum-of-squares function. When a sample yields multiple maximizers of the log-likelihood function, we take $\widehat{\boldsymbol{\theta}}_{n}$ to be the one selected sequentially as follows:

(i) pick out the one(s) with the least vector norm;

(ii) keep the one(s) with the least γ-value;

(iii) if necessary, select that with the least τ -value.

4 - Conditions for the Basic Bent-cable

Given $\delta > 0$ and sequence $\xi_n \downarrow 0$,

$$c_0(\delta) = \liminf_{n\to\infty} \frac{1}{n}\sum 1\{|x_i - \tau_0| \le \gamma_0 + \delta\}$$

$$c_-(\delta) = \liminf_{n\to\infty} \frac{1}{n}\sum 1\{|x_i - \tau_0| \le \gamma_0 - \delta\}$$

$$c_+(\delta) = \liminf_{n\to\infty} \frac{1}{n}\sum 1\{x_i \ge \tau_0 + \gamma_0 + \delta\}$$

$$\zeta(\xi_n) = \frac{1}{n}\sum 1\{\left||x_i - \tau_0| - \gamma_0\right| \le \xi_n\}$$

The regularity conditions on the design are

[A_1] If $\gamma_0 = 0$, then $\forall\ \delta > 0$, $c_0 > 0$

[A_2] If $\gamma_0 > 0$, then $\exists\ \delta_1 > 0$ such that $c_-^* \equiv c_-(\delta_1) > 0$

[B] For $\gamma_0 \ge 0$, $\exists\ \delta_{11} > 0$ such that $c_+^* \equiv c_+(\delta_{11}) > 0$

[C] If $\gamma_0 > 0$, then $\forall \xi_n \downarrow 0, \zeta(\xi_n) \to 0$

[D] If $\gamma_0 > 0$, then $x_i \ne \tau_0 \pm \gamma_0 \forall\ i = 1, ..., n$

In practice, conditions [A] to [D] serve as a set of guidelines for designing the experiment. To reliably estimate the parameters of an underlying basic bent-cable, a non-trivial fraction of observations must be collected from well inside the bend $[\tau_0 - \gamma_0, \tau_0 + \gamma_0]$ in the case of $\gamma_0 > 0$ ([A_2]), and from within all neighborhoods of the break point τ_0 if $\gamma_0 = 0$ ([A_1]). In addition, a non-trivial fraction must come from the outgoing linear phase ([B]). While the true parameters are unknown, the investigator's expertise in the subject matter should nonetheless suggest a range of values which are highly likely inside a certain phase of the model. Condition [C] prevents the accumulation of too many observations exactly at the join points of the underlying bent-cable. Condition [D] is used to avoid technical difficulties in defining the Fisher Information, but it could be eliminated at the cost of some notational complexity. Technicality aside, it is not unreasonable, in the case of a continuous predictor variable, to rule out exact equality of any observed predictor and an underlying join point.

5 -The One-Parameter Basic Bent-cable

Suppose the location of the bend, τ_0, is known. Without loss of generality, take $\tau_0 = 0$. The underlying basic bent-cable is then

$$q(x;\gamma_0)=\frac{(x+\gamma_0)^2}{4\gamma_0}1\{|x|\le\gamma_0\}+x1\{x>\gamma_0\} \tag{3}$$

on the regression domain $x = \mathbb{R}$ and the parameter space $\Omega = [0, \infty)$. Again, we consider maximum likelihood estimation of γ_0 from the data set $\{(x_i, Y_i)\}_{i=1}^{n}$ generated by (1)

The “kinked truth” when the unknown γ_0 is 0, and is a boundary problem — 0 being a boundary value of Ω. This is also an “unidentified model” in the sense of Feder (1975b), in which a segment of the regression model is missing. Feder has shown in Feder (1975b), through an example, that the limiting distribution of the MLE for the unidentified model is not normal.

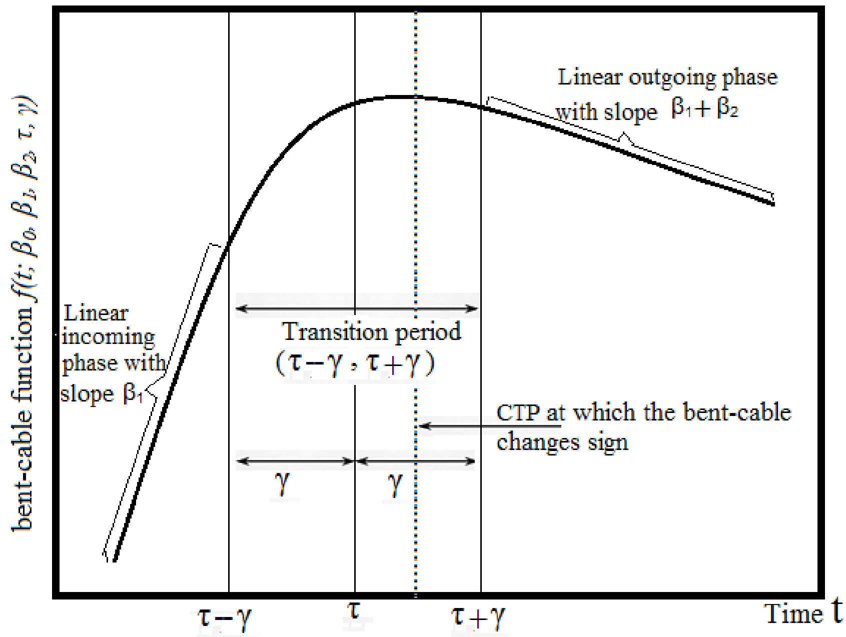
Note also that since $\gamma_0 = 0$ is on the boundary, the cube root asymptotics of Kim and Pollard (1990) are not applicable.

Under this formulation, the transition begins at time $\tau_1 = \tau - \gamma$ and ends at $\tau_2 = \tau + \gamma$, and the critical time point (CTP) at which the slope of the bent-cable changes signs is $\tau-\gamma-\frac{2\beta_1\gamma}{\beta_2}$ (Chiu and Lockhart 2010).

Figure (2)

Graphical description of the bent-cable function

$f(t; \beta_0, \beta_1, \beta_2, \tau, \gamma) = \beta_0 + \beta_1 t + \beta_2 q(t; \tau, \gamma)$



The bent-cable model assumes a quadratic bend (Equation (1)) to characterize the transition zone. In practice, the quadratic function of the bent-cable model may lead to an interval $[\tau_1, \tau_2]$ which is either wider or narrower than what could possibly be necessary to adequately describe the transition zone.

6 -The case study

Climate of Egypt

According to Köppen system (https://www.midat.org), Egypt essentially has a hot desert climate (*BWh*). The climate is generally extremely dry all over the country except on the northern Mediterranean coast which receives rainfall in winter. In addition to rarity of rain, extreme heat during summer months is also a general climate feature of Egypt although daytime temperatures are more moderated along the northern coast.

Prevailing wind

The prevailing northwesterly wind from the Mediterranean Sea continuously blows over the northern coast without the interposition of an eventual mountain range and thus, greatly moderates temperatures throughout the year. Because of the effect, average low temperatures vary from 9.5 °C in wintertime to 23 °C in summertime and average high temperatures vary from 17 °C in wintertime to 32 °C in summertime. Though temperatures are moderated along the coasts, the situation changes in the interior, which are away from the moderating northerly winds. Thus, in the central and the southern parts, nighttime temperatures are very hot, especially in summers where average high temperatures can exceed 40 °C like in Aswan, Luxor, Asyut or Sohag which are located in the deserts of Egypt.

Importance of climate study

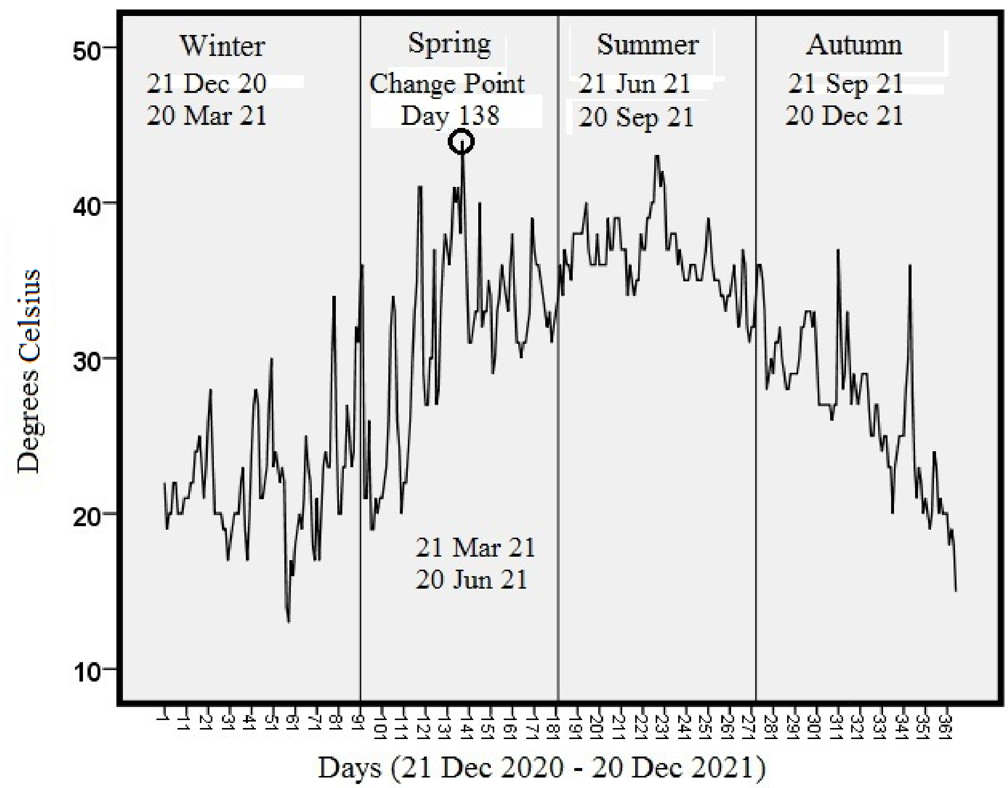
Climate of Egypt, especially the daily temperature, has great effect on two main activities, agriculture and tourism. Many places in Egypt are summer resorts, while some other are winter resorts. In both cases daily temperature is of importance. As for agriculture, all crops are affected by the high (or low) temperature. Hence climate study is of importance, weather climatologically or statistically. By using both traditional and innovative statistical methods, however, we are able to detect and quantify rates of climatic change, estimate the probability of extreme events, and reveal uncertainties in our current understanding of climatic processes and models.

The daily maximum temperature in Cairo through the four seasons of 2021. are collected daily from www. https://accuweather.com. Prospects for 2022 are also collected from the same site. Most of 2022 projections are averages of the previous three years. Data for 2022 are collected to compare the bent-cable projections with the projections published on https://accuweather.com

The following figure (3) presents the daily maximum temperature in Celsius degrees.

Figure (3)

Daily maximum temperature in Cairo



7 -Statistical tests

Using SPSS-20 some statistical tests are performed. Test for randomness shows that the sequence of values for 2021 is not random as shown in the following table (1)

Table (1)
Randomness test

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The sequence of values defined by Temp of 2021<=30.00 and >30.00 is random.	One-Sample Runs Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Applying Kolmogorov-Smirnov test to the data shows that the daily maximum temperature of 2021 does not follow normal distribution.

Table (2)
Test for normal distribution

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Temp of 2021 is normal with mean 29.26 and standard deviation 6.91.	One-Sample Kolmogorov-Smirnov Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Applying the same test shows that the daily maximum temperature of 2021 does not follow Poisson distribution either.

Table (3)
Test for Poisson distribution

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Temp of 2021 is Poisson with mean 29.26.	One-Sample Kolmogorov-Smirnov Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

The tests show that the 2021 temperature are not random and do not follow normal or Poisson distribution, this invites for another statistical method to analysis these data.

8 -Bent-cable estimation

To apply the bent-cable estimation to 2021 data will be divided into three phases shown in figure (3)

Phase I: Linear incoming equation

Table (4)
Incoming linear equation

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	19.391	.834		23.247	.000
	T	.063	.012	.435	5.250	.000

a. Dependent Variable: Y

$$\hat{Y}_1 = 19.391 + 0.063\ t_1$$

Phase II: Transition period

Table (5)
Quadratic estimation of the transition period
Model Summary and Parameter Estimates
Dependent Variable: Y

Equation	Model Summary					Parameter Estimates		
	R Square	F	df1	df2	Sig.	B0	β1	β2
Quadratic	.125	10.763	2	151	.000	32.396	.094	-.001-

The independent variable is t.

Transition period equation estimated as:

$\hat{Y}_2$ = 32.396 + 0.094 t - 0.001 t^2

Phase III: Linear outgoing equation

Table (6)
Outgoing linear equation

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	32.831	.609		53.887	.000
	T	-.135-	.012	-.778-	-11.630-	.000

a. Dependent Variable: Y

$$\hat{Y}_3 = 32.831 - 0.135\, t_3$$

Figure (4)
Three phases of bent-cable regression estimation

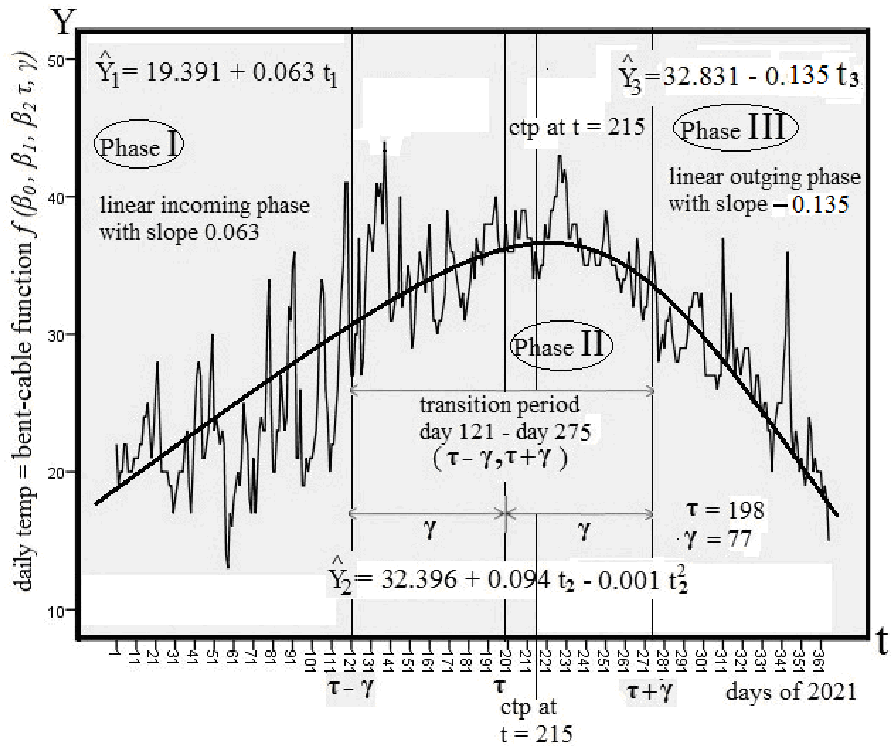
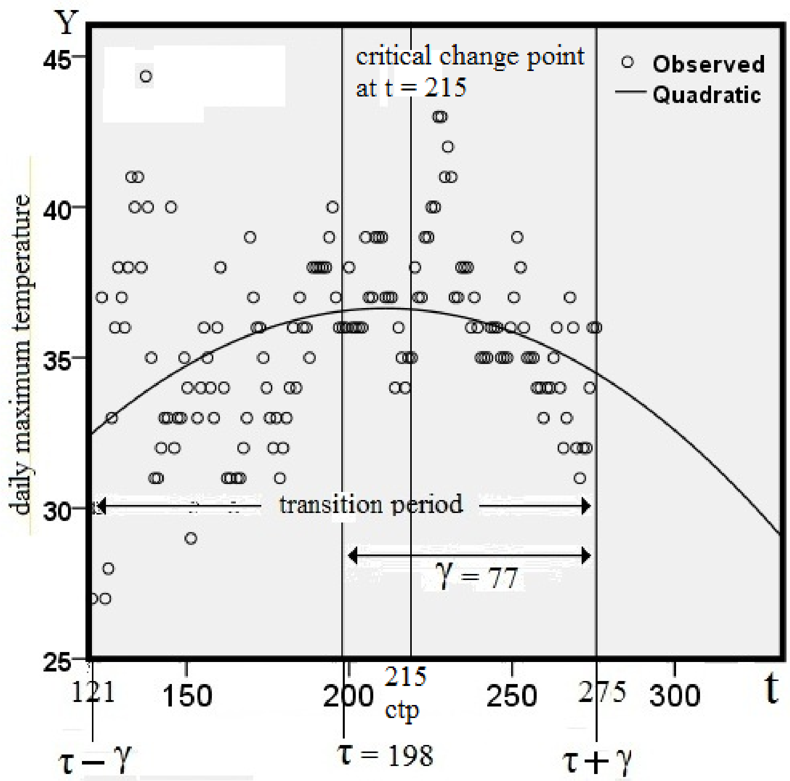


Figure (5)
Quadratic estimation of the transition period



The critical change point (ctp) occurs when the slope of the quadratic equation = 0, i.e $0.094\,t - 0.001\,t^2 = 0 \Rightarrow t = 94$
ctp = $t + \tau - \gamma = 94 + 198 - 77 = 215$

9 -Projections for 2022

The generalized bent-cable regression yields three equations:

1 - Linear incoming phase: from day 1 to day 120

$$\hat{Y}_1 = 19.391 + 0.063\ t_1$$

2 - Transition period quadratic equation: from day 121 to day 275

$$\hat{Y}_2 = 32.396 + 0.094\ t_2 - 0.001\ t_2^2$$

3 - Linear outgoing phase: from day 276 to day 365

$$\hat{Y}_3 = 32.831 - 0.012\ t_3$$

Projections for 2022 presented in the following table compared to those of https://accuweather.com:

Table (7)

Compared projections of maximum temperature in Cairo 2022

P1 = Our projections P2 = accuweather projections

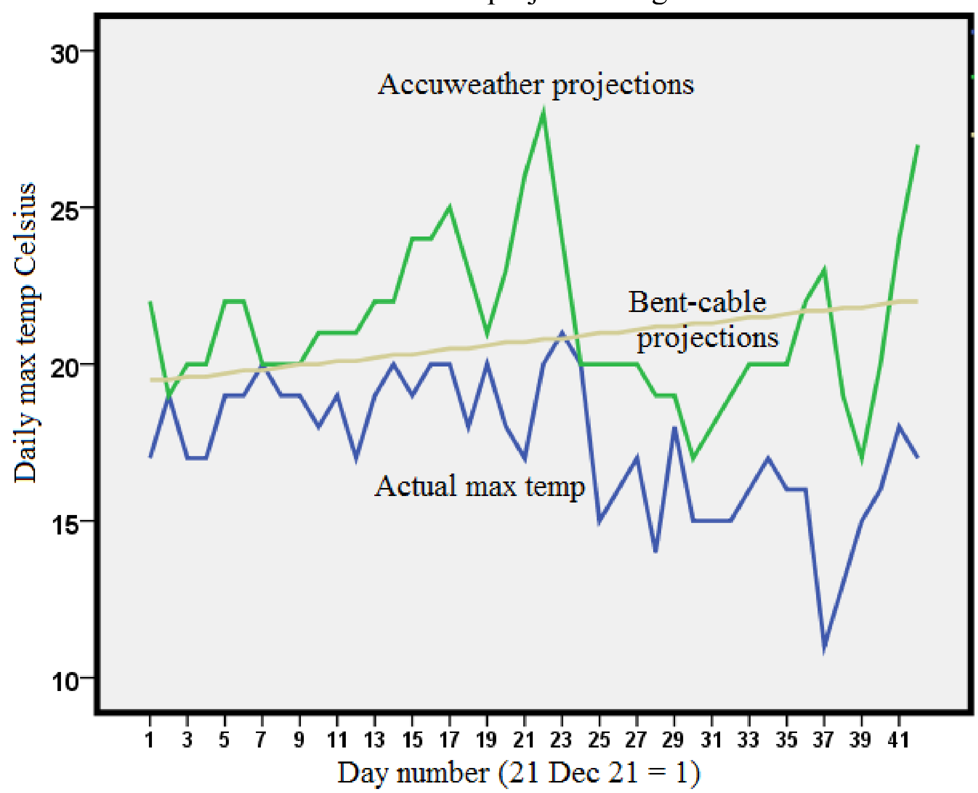
Phase I				Phase II				Phase III			
day	P1	P2	P1-P2	day	P1	P2	P1-P2	day	P1	P2	P1-P2
1	19.5	22	- 2.5	121	32.5	27	5.5	276	32.7	35	- 2.3
20	20.7	23	- 2.3	140	33.9	35	- 1.1	280	32.1	30	2.1
40	22.0	20	2.0	160	34.6	36	- 2.6	300	29.4	33	- 2.6
60	23.2	16	7.2	180	34.4	32	- 2.4	320	26.7	27	- 0.3
80	24.5	25	-.0.5	200	33.5	38	- 4.5	340	24.0	25	- 1.0
100	25.7	21	4.7	220	31.8	38	- 6.2	360	21.3	20	1.3
120	27.0	29	- 2	240	29.3	35	- 5.7	365	20.7	15	- 4.3
				260	26.0	34	- 8				
				275	22.9	36	- 3.1				

9 -Discussion

The accuweather projections are historical averages of the previous three years, i.e 2019, 2020 and 2021 keeping in mind that each year begins on 21 December of the preceding year. Temperature through the above mentioned three years was unusually high so accuweather projections obviously higher than those of 2022.The same case with bent-cable projections which based 2021 data (the high temperature year). So, our

Figure (6)

Accuweather and bent-cable projections against actual data

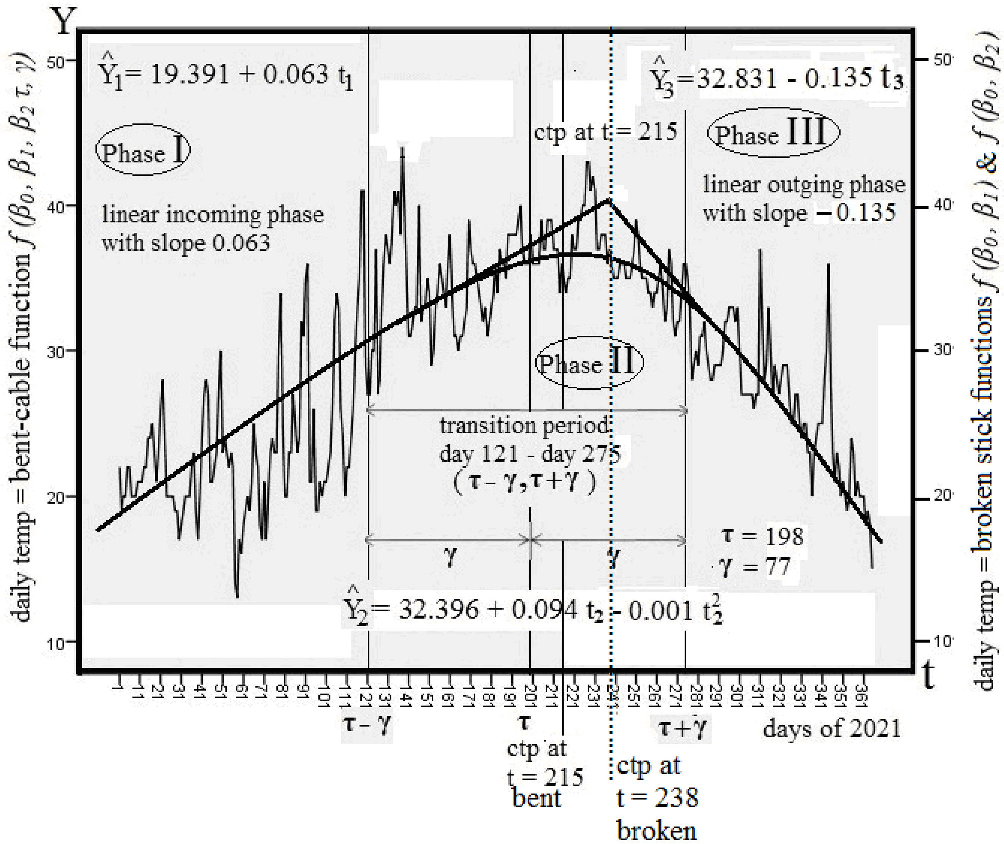


projections are closer to accuweather projections than to the actual data. Both projections are higher than the actual data since the previous three years were abnormally hot years so the projections based on these three years are higher than the actual data. Meanwhile, our analysis is based on the previous 2021 year which was abnormally hot.

10 - Broken stick and bent-cable

The two phases of the broken stick model are shown in the figure (7) against the three phases of the bent-cable model. Before and after the transition period the two models coincide. Hence the difference between the two models is the transition period, since for the broken stick $\gamma = 0$ where in the bent-cable $\gamma = 77$. As a result, the critical time point (CTP) became at $t = 238$ instead of $t = 215$ for the bent-cable. The projections of the two model will differ in the transition period, so we compare the broken stick projections against accuweather projections.

Figure (7)
Broken stick and bent-cable functions



The following table (8) summarizes this comparison. The broken stick projections are above those of the bent-cable as shown on figure (7).

Table (8)
Comparison of the transition period projections
P1 = bent P2 = accuweather P3 = broken

day	P1	P2	P3	P3-P1	P3-P2
121	32.5	27	33.9	1.4	6.9
140	33.9	35	38.8	4.9	3.8
160	34.6	36	37.7	3.1	1.7
180	34.4	32	36.5	2.1	4.5
200	33.5	38	35.3	1.8	- 2.7
220	31.8	38	34.1	2.3	- 3.9
240	29.3	35	32.9	3.6	- 2.1
260	26.0	34	31.7	5.2	- 2.3
275	22.9	36	29.5	6.6	- 6.5

It is worth noting that all projections are far from the actual of data of the first 50 days of 2022. This due to the fact that the previous three years where all projections are based on.

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تطبيق انحدار الكابل المنحني على درجات الحرارة العظمى اليومية في القاهرة خلال ٢٠٢١

د. سمر أحمد حلمي؛ د. سماح كمال

قسم الإحصاء - كلية التجارة - جامعة بورسعيد

في هذا البحث سنستخدم نموذج انحدار الكابل المنحني لتحليل درجات الحرارة العظمى اليومية في مدينة القاهرة خلال عام ٢٠٢١. يقوم هذا النموذج على جزأين خطيين، متصلين بسلاسة من خلال منحنى تربيعي. وتشتمل فئة الكابلات المنحنية في النهاية على النموذج الخطي المجزأ الشائع (مع التواء حاد)، والمعروف أيضًا باسم العصا المكسورة. ويرتبط بانحدار الكبل المنحني تقدير معامل عرض الانحناء، والذي يمكن من خلاله تقييم الانقطاع المفاجئ للانتقال الأساسي. استخدمت درجة الحرارة العظمى المسجلة في القاهرة لإثبات انتظام وعدم انتظام انحدار الكابلات المنحنية المصادفة في إعدادات العينات المحدودة. قدمنا أيضًا تنبؤات الكبل المنحني لعام ٢٠٢٢ مقارنةً بتنبؤات العصا المكسورة وموقع دراسات الطقس accuweather. وقد تم عرض الشروط العملية اللازمة لضمان انتظام تقدير الكبل المنحني بالكامل، إذا كان عرض مقطع الانحناء الأساسي غير صفري. في ظل هذه الظروف، تظهر مقدرات المربعات الصغرى (١) لتكون متسقة و (٢) لتتبع توزيعًا طبيعيًا متعدد المتغيرات بشكل مقارب. ومن ثم تُستخدم مقدرات المربعات الصغرى لتحليل البيانات المدروسة.

وكما هو معروف فإن توقعات Accue.kijl, mather هي متوسطات تاريخية للسنوات الثلاث السابقة، أي ٢٠١٩ و٢٠٢٠ و٢٠٢١ مع الأخذ في الاعتبار أن كل عام يبدأ في ٢١ ديسمبر من العام السابق. كانت درجة الحرارة خلال السنوات الثلاث المذكورة أعلاه مرتفعة بشكل غير عادي، لذا من الواضح أن توقعات Accueather أعلى من البيانات الفعلية لعام ٢٠٢٢، نفس الحالة مع تنبؤات الكابلات المنحنية التي تستند إلى بيانات عام ٢٠٢١ (عام درجة الحرارة المرتفعة). ومع ذلك كانت تنبؤاتنا أقرب للبيانات الفعلية لعام ٢٠٢٢ من تنبؤات Accueather.

كلمات مفتاحية:

كابل منحني - عصا مكسورة - تقسيم مناخي - مقدرات الإمكان الأعظم - نقطة التغير