The New Extension of Inverse Weibull Distribution with Applications of Medicine Data

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Scientific Journal for Financial and Commercial Studies and Researches (SJFCSR)

Faculty of Commerce – Damietta University

Vol.2, No.1, Part 1., Jan. 2021

APA Citation:

Ibrahim, G. M**.** and **Almetwally**, E. M. (2021). The New Extension of Inverse Weibull Distribution with Applications of Medicine Data**.** ***Scientific Journal for Financial and Commercial Studies and Researches (SJFCSR)***, Vol.2 (1) Part1. pp.577- 598.

Website: https://cfdj.journals.ekb.eg/

Abstract:

This paper introduced and studied a new extension of inverse Weibull distribution with three-parameter named as the X-Gamma inverse Weibull (XGIW) distribution. Reliability and hazard rate properties of this distribution are discussed. Maximum likelihood estimation (MLE), maximum product spacing (MPS) Method of the XGIW distribution parameters are discussed. A numerical study using real data analysis and Monte-Carlo simulation are performed to compare between MLE and MPS methods of estimation. The flexibility and potentiality of the XGIW distribution are examined using two real data sets. The cancer data represents remission times (in months) of a random sample of 128 bladder cancer patients and the second data set of leukemia represents 40 patient suffering from leukemia from one of the Ministry of Health Hospitals in Saudi Arabia. The XGIW model can produce better fits than some well-known distributions as generalized inverse Weibull, Kumaraswamy–inverse Weibull, exponentiated generalized inverse Weibull distribution and inverse Weibull distributions.

Keywords: X-Gamma family; inverse Weibull distribution; maximum likelihood estimation; maximum product spacing; data analysis.

1. Introduction

Keller and Kamath (1982) discussed the shapes of the density and failure rate functions for the basic inverse distribution. The inverse Weibull (IW) distribution has received some attention in more literature it can be used in the reliability engineering discipline and to model a variety of failure characteristics such as infant mortality, useful life and wear-out periods see Khan et al. (2008). The cumulative distribution function (CDF) and the probability density function (PDF) of the IW distribution are respectively as follows

$$F(x;\lambda,\alpha)=e^{-\lambda x^{-\alpha}} \ ; \ ,x>0;\ \lambda,\alpha>0, \qquad (1.1)$$

$$f(x;\lambda,\alpha)=\alpha\,\lambda\,x^{-\alpha-1}\,e^{-\lambda x^{-\alpha}}. \qquad (1.2)$$

Modified and extended versions of the IW distribution have been studied by many authors as Shahbaz et al. (2012) who presented a new extension of the IW distribution based Kumaraswamy family. Khan et al. (2013) studied the transmuted inverse Weibull distribution. De Gusmao et al. (2011) introduced generalized inverse Weibull distribution by using exponential family. Baharith et al. (2014) discussed the beta generalized inverse Weibull distribution using beta family. El batal and Muhammad (2014) introduced an exponentiated generalized inverse Weibull distribution by using exponentiated generalized family. Basheer (2019) introduced Alpha power inverse Weibull distribution. Basheer et al. (2020) introduced Marshall-Olkin alpha power inverse Weibull.

During recent years, the X-Gamma (XG) distribution has been taken a great interest by researchers, which is introduced by Sen et al (2016). The quasi-XG distribution has been introduced by Sen and Chandra (2017). A new bounded distribution has been introduced by Altun and Hamedani (2018) using the transformation $Y = e^{-X}$ as an alternative to the beta distribution based on the XG distribution. Another generalization of XG distribution has been provided by Sen et al. (2018_a) on the basis of a special mixture of exponential and gamma distributions. Parameter estimation of XG distribution under the progressively type-II censored sample has been studied by Sen et al. (2018_b) by using different methods. The transmuted-XG distribution has been studied by Biçer (2019). Yadav et al. (2019) introduced inverse X-Gamma distribution using the transformation $Y = \frac{1}{X}$. Bantan et al. (2020) introduced the half-logistic XG distribution using half-logistic family. Sen et al. (2020) studied discrimination analysis between the Lindley and XG distribution.

On the other hand, the XG-Generator (XG-G) family has been proposed by Cordeiro et al. (2019) to incorporate any distribution into a larger family. The XG-G family has flexible shapes to model various lifetime data sets. The XG-G family added a parameter which has one extra shape parameter $\vartheta > 0$,the CDF of XG-G family is given by

$$F(x;\vartheta,\psi) = 1 - \frac{[1 - G(x;\psi)]^{\vartheta}}{\vartheta + 1}\left\{1 + \vartheta - \vartheta \ln\big(1 - G(x;\psi)\big) + 0.5\vartheta^{2}\big[\ln\big(1 - G(x;\psi)\big)\big]^{2}\right\}, \quad (1.3)$$

where $G(x;\psi)$ is a baseline CDF with a parameter vectorψ. The PDF of XG-G family can be expressed as:

$$f(x;\vartheta,\psi) = \frac{\vartheta}{\vartheta+1} g(x;\psi)[1-G(x;\psi)]^{\vartheta-1}\Big\{\vartheta + 0.5\vartheta^2\big[\ln\big(1-G(x;\psi)\big)\big]^2\Big\} \tag{1.4}$$

where $g(x;\psi) = dG(x;\psi)/dx$.

The aim of this paper is to clarify two aspects. Firstly, propose and study a new lifetime distribution called X-Gamma inverse Weibull (XGIW) distribution based on the XG-G family. Reliability and hazard rate properties of XGIW distribution are provided. Secondly, parameters estimation of the XGIW distribution are discussed by using MLE and MPS methods. To evaluate the performance of the estimators, extensive simulation study is carried out. Our XGIW model as well as some other well-known distributions are illustrated by two real data sets. The XGIW distribution can produce better fits than some well-known distributions.

The paper is organized as follows; in section 2, we introduce the description and notation of XGIW distribution, while in section 3 reliability and hazard rate properties of XGIW distribution are discussed. In section 4 we discuss the parameter estimation of XGIW distribution. In section 5, Monte-Carlo simulation study are presented to compare the performance of the parameter's estimation for different methods. In section 6, applications of three real data sets are studied. Finally, in section 7, we discuss the results and conclusions of the current study.

2. Model Description and Notation

In this section, we will introduce the XGIW distribution and some of its sub-models. The XG-G family and IW distribution have been used to generate XGIW distribution. It is represented by the random variable $X \sim XGIW\ (\vartheta,\lambda,\alpha)$. By using Equations (1.3, 1.4, 1.1 and 1.2), the CDF of XGIW distribution takes this form:

$$F(x;\vartheta,\lambda,\alpha) = 1 - \frac{\left[1 - e^{-\lambda x^{-\alpha}}\right]^{\vartheta}}{\vartheta + 1}\left\{1 + \vartheta - \vartheta \ln\left(1 - e^{-\lambda x^{-\alpha}}\right) + 0.5\vartheta^{2}\left[\ln\left(1 - e^{-\lambda x^{-\alpha}}\right)\right]^{2}\right\}, \qquad (2.1)$$

where $\vartheta, \lambda, \alpha > 0$ and $x > 0$. The PDF of XGIW distribution is given as:

$$f(x,\vartheta,\lambda,\alpha) = \frac{\vartheta\alpha\,\lambda}{\vartheta + 1}\, x^{-\alpha-1}\, e^{-\lambda x^{-\alpha}}\left[1 - e^{-\lambda x^{-\alpha}}\right]^{\vartheta-1}\left\{\vartheta + 0.5\vartheta^{2}\left[\ln\left(1 - e^{-\lambda x^{-\alpha}}\right)\right]^{2}\right\} \qquad (2.2)$$

Figure 1 display plots of the PDF of the XGIW distribution for some parameters values as follows:

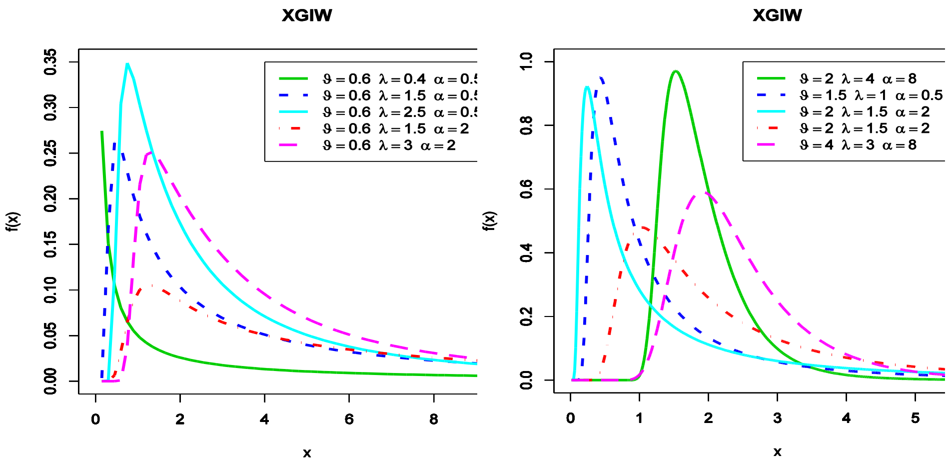

Figure 1. Plots of the PDF of the XGIW distribution for Some Values of Parameters.

Therefore, for a random variable X~XGIW $(\vartheta, \lambda, \alpha)$ with PDF (2.2), we have the following cases:

1. If $\alpha \to 1$, then PDF (2.2) reduces to the two-parameter distribution, this is a new model which can be denoted as X-Gamma inverse exponential (XGIEx) distribution.
2. If $\alpha \to 2$, then PDF (2.2) reduces to the two-parameter distribution, this is a new model which can be denoted as X-Gamma inverse Rayleigh (XGIR) distribution when $\alpha \to 2$.

3. Reliability Analysis of XGIW Distribution

The survival function of XGIW distribution is given by

$$S(x;\vartheta,\lambda,\alpha) = \frac{\left[1-e^{-\lambda x^{-\alpha}}\right]^{\vartheta}}{\vartheta+1}\left\{1+\vartheta-\vartheta\ln\left(1-e^{-\lambda x^{-\alpha}}\right) + 0.5\vartheta^{2}\left[\ln\left(1-e^{-\lambda x^{-\alpha}}\right)\right]^{2}\right\} \tag{3.1}$$

The hazard rate function of a lifetime random variable *X* with XGIW distribution is given by:

$$h(x;\vartheta,\lambda,\alpha) = \frac{\vartheta\alpha\,\lambda\,x^{-\alpha-1}\,e^{-\lambda x^{-\alpha}}\left\{\vartheta+0.5\vartheta^{2}\left[\ln\left(1-e^{-\lambda x^{-\alpha}}\right)\right]^{2}\right\}}{\left(1-e^{-\lambda x^{-\alpha}}\right)\left\{1+\vartheta-\vartheta\ln\left(1-e^{-\lambda x^{-\alpha}}\right)+0.5\vartheta^{2}\left[\ln\left(1-e^{-\lambda\right.\right.}\right.} \quad (3.2)$$

Figure 2 display plots of the hazard function of the XGIW distribution for some values of the parameters as follows:

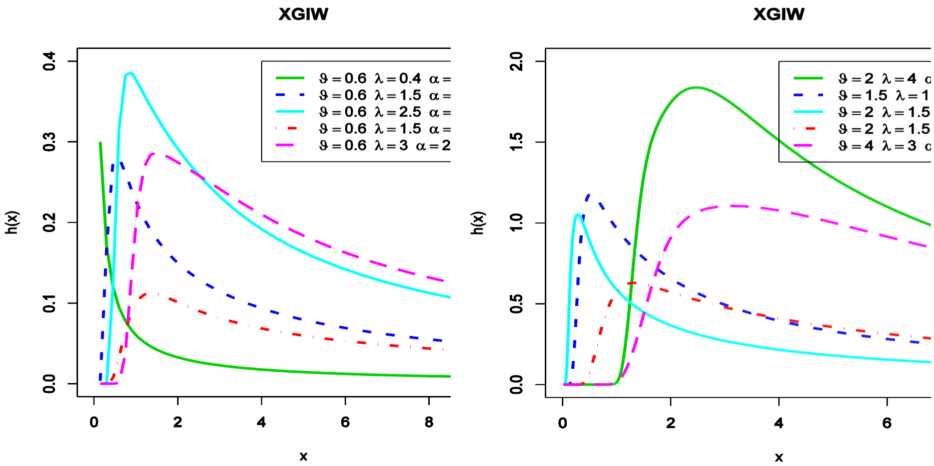

Figure 2. Plots of the hazard of the XGIW with Some Values of the Parameters.

4. Parameter Estimation

In this section, the parameter estimation of the XGIW distribution using MLE and MPS estimation methods in the presence of complete sample will be discussed in details.

4.1. MLE method

The log-likelihood function of XGIW distribution, is given by:

$$l(\vartheta,\alpha,\lambda) = n\,ln\left(\frac{\vartheta}{\vartheta+1}\right) + n[\ln(\alpha)+\ln(\lambda)] - (\alpha+1)\sum_{i=1}^{n}\ln(x_i) - \lambda\sum_{i=1}^{n}{x_i}^{-\alpha} + (\vartheta-1)\sum_{i=1}^{n}\ln\left(1-e^{-\lambda {x_i}^{-\alpha}}\right) + \sum_{i=1}^{n} ln\left\{\vartheta + 0.5\vartheta^2\left[\ln\left(1-e^{-\lambda {x_i}^{-\alpha}}\right)\right]^2\right\}. \tag{4.1}$$

Equation (4.1) can be maximized directly by using the R package by an optim function, to solve the non-linear likelihood equations obtained by differentiating Equation (4.1) with respect to $\vartheta, \alpha, \lambda$ and equating to zero. The non-linear likelihood equations are given as:

$$\frac{\partial l(\vartheta,\alpha,\lambda)}{\partial\vartheta} = \frac{n}{\vartheta(\vartheta+1)} + \sum_{i=1}^{n}\ln\left(1-e^{-\lambda {x_i}^{-\alpha}}\right) + \sum_{i=1}^{n}\frac{1+\vartheta\left[\ln\left(1-e^{-\lambda {x_i}^{-\alpha}}\right)\right]^2}{\vartheta + 0.5\vartheta^2[\ln(1-e^{-\lambda {x_i}^{-\alpha}})]^2},$$

$$\frac{\partial l(\vartheta,\alpha,\lambda)}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\ln(x_i) + \lambda\sum_{i=1}^{n}{x_i}^{-\alpha}\ln(x_i) - (\vartheta-1)\lambda\sum_{i=1}^{n}\frac{e^{-\lambda {x_i}^{-\alpha}}{x_i}^{-\alpha}\ln(x_i)}{1-e^{-\lambda {x_i}^{-\alpha}}} - \vartheta^2\sum_{i=1}^{n}\frac{\ln\left(1-e^{-\lambda {x_i}^{-\alpha}}\right)\frac{e^{-\lambda {x_i}^{-\alpha}}{x_i}^{-\alpha}\ln(x_i)}{1-e^{-\lambda {x_i}^{-\alpha}}}}{\vartheta + 0.5\vartheta^2[\ln(1-e^{-\lambda {x_i}^{-\alpha}})]^2},$$

and

$$\frac{\partial l(\vartheta,\alpha,\lambda)}{\partial \lambda} = \frac{n}{\omega} - \sum_{i=1}^{n} x_i{}^{-\alpha} + (\vartheta - 1)\sum_{i=1}^{n} \frac{e^{-\lambda x_i{}^{-\alpha}}\, x_i{}^{-\alpha}}{1 - e^{-\lambda x_i{}^{-\alpha}}} + \vartheta^2 \sum_{i=1}^{n} \frac{\ln\left(1 - e^{-\lambda x_i{}^{-\alpha}}\right)\frac{e^{-\lambda x_i{}^{-\alpha}}\, x_i{}^{-\alpha}}{1-e^{-\lambda x_i{}^{-\alpha}}}}{\vartheta + 0.5\vartheta^2[\ln(1 - e^{-\lambda x_i{}^{-\alpha}})]^2}.$$

4.2.MPS Method

MPS method is used to estimate the parameters of continuous univariate models as an alternative to the MLE method. The uniform spacings of a random sample $x_1 < \cdots < x_n$ of size *n* from the XGIW distribution can be defined by:

$$D_i(\vartheta,\alpha,\lambda) = F(x_i,\vartheta,\alpha,\lambda) - F(x_{i-1},\vartheta,\alpha,\lambda); i = 1,2,\ldots,n+1$$

where D_i refers to the uniform spacings and $\sum_{i=1}^{n+1} D_i = 1$. The MPS estimators can be obtained by maximizing

$$G(\vartheta,\alpha,\lambda) = \frac{1}{n+1}\sum_{i=1}^{n+1} \ln\left(D_i(\vartheta,\alpha,\lambda)\right)$$

For more information of MPS method, see Cheng and Amin (1983), Almetwally and Almongy (2019$_{b,\ a}$), Almetwally et al. (2019, 2020), El-Sherpieny et al. (2020) and Ahmad and Almetwally (2020).

The natural logarithm of the product spacing function for the MPS of XGIW distribution is given by:

$$\ln G(\vartheta,\alpha,\lambda)=\frac{1}{n+1}\Bigg(\ln(\mathscr{b}[\vartheta,\varphi(x_n,\alpha,\lambda)]) + \ln(1-\mathscr{b}[\vartheta,\varphi(x_1,\alpha,\lambda)]) + \sum_{i=2}^{n}\ln(\mathscr{b}[\vartheta,\varphi(x_{i-1},\alpha,\lambda)] - \mathscr{b}[\vartheta,\varphi(x_i,\alpha,\lambda)])\Bigg), \tag{4.2}$$

where $\varphi(x_i,\alpha,\lambda)=1-e^{-\lambda x_i{}^{-\alpha}}$, $\mathscr{b}(\vartheta,\varphi)=\frac{(\varphi)^{\vartheta}}{\vartheta+1}\{1+\vartheta-\vartheta\ln(\varphi)+0.5\vartheta^2[\ln(\varphi)]^2\}$,

The partial derivatives of MPS with respect to the unknown parameters cannot be solved explicitly, so numerical methods like the conjugate gradients algorithms can be used to calculate the MPS of ϑ,α,λ.

5. Simulation Study

In this section; a Monte Carlo simulation is done to estimate the parameters based on complete sample by using MLE and MPS methods. Using R packages and using the following:

Simulation algorithm: Monte Carlo experiments were carried out based on10,000 random sample for following data generated form XGIW distribution by using numerical analysis, where x_i is distributed as XGIW distribution for different parameters $(\vartheta,\alpha,\lambda)$ with different actual values of parameter and for different samples sizes $n\ =50,100\ and\ 200$. We can find the parameter estimation by using Equations (4.1 and 4.2) and using R package. We can define the best method as which minimizes the Bias and root mean squared error (RMSE) of the estimator.

The following conclusions can be drawn from Table (1):

1. All the estimates reveal the property of consistency, i.e., the Bias and RMSE decrease when n increase.
2. The MPS estimates has more relative efficiency than MLE for most parameters of XGIW distribution.

Table 1: MLE and MPS estimation methods with different values of parameters

ϑ				1.5				3			
				MLE		MPS		MLE		MPS	
α	λ	n		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
0.6	0.7	50	ϑ	0.2869	0.9705	0.2689	0.4739	-0.6352	1.1262	-0.5570	0.8875
			α	1.1419	1.2391	0.9552	1.0628	1.1219	2.2182	1.1820	1.5535
			λ	-0.1209	0.4230	-0.0929	0.1063	0.1657	0.4013	0.2301	0.1541
		100	ϑ	0.2320	0.7880	0.2347	0.3802	-0.9511	1.0200	-0.5479	0.7519
			α	1.0989	1.1776	0.9572	1.0432	0.9278	1.5345	1.0849	1.1577
			λ	-0.1308	0.3692	-0.1047	0.0936	0.0371	0.3140	0.2051	0.1100
		200	ϑ	0.1197	0.4624	0.2116	0.1500	-0.8957	0.8200	-0.5252	0.7455
			α	1.0787	1.1199	0.9364	0.9191	0.9928	1.3494	0.9854	1.0060
			λ	-0.1780	0.2845	-0.1172	0.0407	0.0346	0.3109	0.2027	0.1085
	1.7	50	ϑ	0.3926	1.0652	0.4932	0.7106	-0.5900	1.1587	-0.4707	0.8256
			α	1.0792	1.1794	0.8233	0.8035	2.1176	2.2270	1.7639	2.1343
			λ	-1.0739	1.1522	-0.9826	1.0668	-0.8218	0.9063	-0.7322	0.6394
		100	ϑ	0.3920	1.0433	0.4970	0.7009	-0.9226	1.0187	-0.5687	0.6716
			α	1.0662	1.1620	0.8190	0.7898	2.2545	2.1322	1.7944	1.7269
			λ	-1.0731	1.1485	-0.9819	1.0630	-0.9509	0.9002	-0.7059	0.6046
		200	ϑ	0.3286	0.5799	0.4196	0.3507	-0.9310	0.9185	-0.5761	0.6761
			α	0.9169	0.9640	0.8122	0.7070	2.2591	1.6326	1.8002	3.3944

			λ	-1.0535	1.0772	-1.0031	1.0487	-0.9415	0.8500	-0.7626	0.6500
1.6	0.7	50	ϑ	-0.0566	0.7825	0.0284	0.4673	-0.7734	1.8874	-0.6091	1.2952
			α	0.5623	0.9789	0.2154	0.3460	1.1765	1.7174	0.9090	1.2953
			λ	0.5633	0.8593	0.6104	0.6694	1.0820	1.4406	1.0210	1.1821
		100	ϑ	-0.0534	0.7281	0.0111	0.4134	-0.7639	1.7900	-0.5985	1.2562
			α	0.5119	0.9362	0.2145	0.3256	1.0142	1.6692	0.8998	1.2673
			λ	0.5272	0.8267	0.6046	0.6450	1.0644	1.4236	0.9208	1.2803
		200	ϑ	-0.0208	0.5602	-0.0081	0.1867	-1.0506	1.5814	-0.7529	1.0809
			α	0.2896	0.5726	0.1587	0.1695	0.8905	1.3082	0.9773	1.2574
			λ	0.5162	0.7862	0.6343	0.5492	0.9389	1.1597	0.9017	1.0572
	1.7	50	ϑ	0.0817	0.7951	0.1896	0.5925	-1.0347	1.5667	-0.6388	0.9425
			α	0.3838	0.8396	0.0848	0.2743	1.7824	2.0607	0.8888	1.0631
			λ	-0.2724	0.6590	-0.2309	0.3245	-0.0278	0.6740	0.2546	0.2563
		100	ϑ	-0.0710	0.7252	0.1254	0.3440	-1.0704	1.4699	-0.6823	0.9787
			α	0.3491	0.8237	0.0761	0.1921	1.6815	1.9042	0.8763	0.9401
			λ	-0.2535	0.6051	-0.2570	0.2864	-0.0249	0.6580	0.2808	0.2321
		200	ϑ	-0.0679	0.6800	0.0830	0.1663	-0.9513	1.4581	-0.6838	0.8410
			α	0.3173	0.6653	0.0591	0.1095	1.5732	1.7833	0.9018	1.0321
			λ	-0.2465	0.5794	-0.2262	0.1840	-0.0640	0.5798	0.2482	0.2022

6. Application of Real Data Analysis

In this section, the flexibility and potentiality of the XGIW distribution are examined using two real data sets. We provide an application of the XGIW distribution and their sub-models: generalized inverse Weibull distribution (De Gusmao et al. (2011)), Kumaraswamy–Inverse Weibull distribution (Shahbaz et al. (2012)), exponentiated generalized inverse Weibull distribution (Elbatal and Muhammed (2014)) and inverse Weibull distribution.

6.1.Cancer Data:

The cancer data set are given by Lee and Wang (2003) which represent remission times (in months) of a random sample of 128 bladder cancer patients. The data is as follows: 0.08, 2.09, 3.48, 4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69.

Table 2. MLE, K–S Distance, P-values, AIC, CAIC, BIC and HQIC with Different Models for the Cancer Data Set.

	ω	ϑ	α	λ	KS	P-Value	AIC	CAIC	BIC	HQIC
IW			0.7521	2.4311	0.140	0.0125	892.0015	892.0975	897.7056	894.3191
			0.0424	0.2193						
XGIW		162.065	0.2019	7.8440	0.054	0.8505	828.7930	828.9866	837.3491	832.2694
		749.799	0.1725	4.4204						
KIW		0.2103	7.6985	132.0639	0.049	0.9158	828.9439	829.1374	837.5000	832.4202
		0.0372	0.8915	118.3662						
GIW		1.3169	1.9765	0.7521	0.124	0.0825	894.0015	894.1951	902.5576	897.4779
		130.743	147.5866	0.0424						
EGIW	93.112	11.132	0.4172	0.6943	0.066	0.6306	839.9249	840.2502	851.3331	844.5601
	33.111	2.878	0.1222	0.6481						

In table 2, the XGIW model has the highest p-value and the lowest distance (D) of Kolmogorov–Smirnov (K-S) value when it compares with all other models that used here to fit the current physical data. Table 2 shows that the new model (XGIW) fits the data better than the IW, GIW, KIW and EGIW models based on these different criterions as the Akaike information criterion (AIC), correct Akaike information criterion (CAIC), Bayesian information criterion (BIC) and Hannan–Quinn information criterion (HQIC) values. Figure 3 shows the fit of the empirical CDF, histogram and PP-plot as follows:

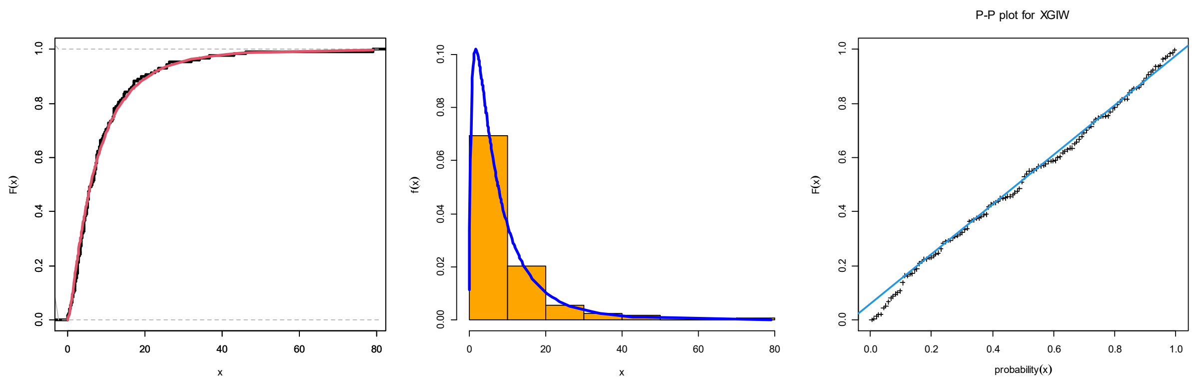

Figure 3. Cumulative function and empirical CDF, histogram and the Fitted MOAPL distribution, Q-Q plot and P-P plot for the MOAPL distribution for the physical data set.

6.2. Leukemia Data:

The second data set are given by Abouammoh et al. (1994) which represent 40 patients suffering from leukemia from one of the Ministry of Health Hospitals in Saudi Arabia. The data are recorded as follows 115 ,181, 255, 418, 441, 461, 516, 739, 743 ,789 ,807, 865, 924, 983, 1024,1062, 1063,1165, 1191,1222,1251,1277, 1290 ,1357,1369, 1408 ,1455, 1478, 1222, 1549, 1578, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815 ,1852.

Table 3. MLE, K–S Distance and P-values, AIC, CAIC, BIC and HQIC with different models for the leukemia data Set.

	ω	ϑ	α	λ	KS	P-Value	AIC	CAIC	BIC	HQIC
IW			0.7750	152.0229	0.359	0.0001	669.2719	669.5963	672.6497	670.4932
			0.0578	53.9062						
XGIW		206.1569	0.3652	71.4588	0.165	0.2957	623.5790	626.0246	630.1646	627.0411
		170.9943	0.0596	23.6786						
KIW		0.3763	76.8584	198.0307	0.169	0.2029	625.5534	626.2201	630.6200	627.3853
		0.0680	28.1455	186.9563						
GIW		30.4337	41.7948	1.1984	0.262	0.0082	657.2778	657.9445	662.3445	659.1098
		26.8457	19.6703	0.1242						
EGIW	30.410	0.3533	3.3064	36.7333	0.261	0.0083	660.5808	661.7237	667.3364	663.0234
	26.552	0.2645	1.6459	42.1134						

In table 3, the XGIW model has the highest p-value and the lowest distance of K-S value when it compares with all other models that used here to fit the leukemia data. Table 3 shows that the XGIW fits the data better than the IW, GIW, KIW and EGIW models based on different criteria as the AIC, CAIC, BIC and HQIC values. Figure 4 shows the fit of the empirical CDF, histogram and PP-plot as follows.

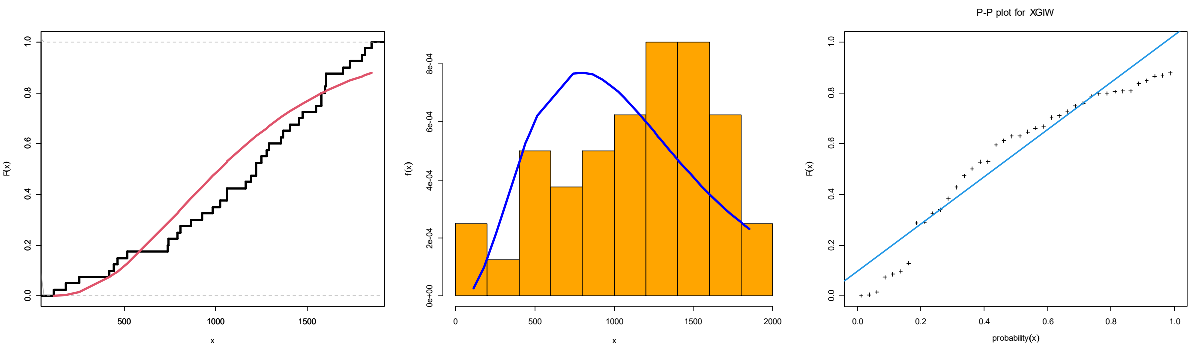

Figure 4. Cumulative function and empirical CDF, histogram of the Fitted XGIW distribution and P-P plot for the XGIW distribution for the leukemia data set.

7. Conclusion

In this paper, we propose a new four-parameter model, called the X-Gamma inverse Weibull (XGIW) distribution, which is a new extension of the inverse Weibull distribution. The XGIW distribution is motivated by the wide utilization of the inverse Weibull model in life testing and provides more flexibility to analyze lifetime data. The parameter estimation of XGIW distribution is derived by MLE and MPS. The methods of estimation are employed to estimate the model parameters and simulation results are provided to assess the model performance. Three real-life data proposed model provides a consistently better fit than the IW, GIW, KIW and EGIW distributions.

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تعميم جديد لتوزيع مقلوب وايبل بالتطبيق على بيانات طبية

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الملخص:

في هذا البحث تم تقديم تطوير جديد لتوزيع inverse Weibull، حيث يتم ادخال توزيع inverse Weibull علي عائلة X-Gamma والحصول علي توزيع ذي ثلاثة معلمات وهو توزيع أكس- جاما وايبل X-Gamma inverse Weibull ويمكن ان طلق عليه الاختصار التالي (XGIW)، حيث تم مناقشة Reliability وخصائص hazard rate لهذا التوزيع. تم استخدام طريقة الامكان الأعظم Maximum likelihood estimation (MLE) وطريقة maximum product spacing (MPS) لتقدير معالم التوزيع المقترح.

هدف البحث الحصول على نموذج أكثر ملائمة للبيانات والمقارنة بين كفاءة الأساليب المختلفة في تقدير معالم التوزيع المقترح، حيث تم إجراء تحليل لبيانات رقمية باستخدام بيانات حقيقية، ثم بيانات محاكاه تم الحصول عليها باستخدام طريقة مونت-كارلو، وذلك للمقارنة بين تقديرات كل من طريقة الإمكان الأعظم وطريقة MPS. كذلك تم تقييم مرونة واعتمادية توزيع (XGIW) المقترح من خلال مجموعتين من البيانات الحقيقية والتي تمثل بيانات عن مدة البقاء بالأشهر لعينة عشوائية بلغت ١٢٨ من مرضى سرطان المثانة. أما مجموعة البيانات الأخرى فهي عبارة عن بيانات عينة تبلغ ٤٠ من مرضى سرطان الدم تم أخذها من أحد مستشفيات وزارة الصحة في المملكة العربية السعودية. حيث ثبت أن تقديرات التوزيع المقترح كانت أفضل من بعض التوزيعات المعروفة مثل توزيع generalized inverse Weibull، وتوزيع Kumaraswamy–inverse Weibull، وكذلك توزيع exponentiated generalized inverse Weibull، inverse Weibull.

الكلمات المفتاحية: عائلة X-Gamma، توزيع inverse Weibull، طريقة الامكان الأعظم، maximum product spacing، تحليل البيانات